Arithmetic sequences:	**Not** arithmetic sequences:
1, −2, −5, −8, …	1, −2, −6, −11, …
16, 14, 12, 10, …	16, 14, 10, 2, …
6, 16, 26, 36, …	6, 16, 11, 21, 16, 26, 21, …

Define what it means to be an arithmetic sequence in your own words. Give an example.

Geometric sequences:	**Not** geometric sequences:
1, 2, 4, 8, …	1, 2, 6, 24, …
100, 50, 25, 12.5, …	600, 300, 100, 25, …
3, −12, 48, −192, …	3, −12, 60, −360, …
$\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, ...$	$\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, ...$

Define what it means to be a geometric sequence in your own words. Give an example.

Definition: Arithmetic Sequence (Alg 2 Review)

When the difference between successive terms is always the same. This difference is called the ***common difference***, and is denoted by the letter ***d***.

Formula for *nth* Term of an Arithmetic Sequence

$a_n = a + (n - 1)d$ or $a_n = a + d(n - 1)$

where $a = a_1$

Example 1: Find the 13^{th} term of the sequence: 2, 6, 10, 14, …

Step 1 Determine a_1 and ***d*** $a_1 = 2$ $\boldsymbol{d} = 4$

Step 2 Write formula, substitute in values, and simplify

$a_n = a + d(n - 1)$

$a_n = 2 + 4(n - 1)$ → This is the general formula

$a_{13} = 2 + 4(13 - 1)$

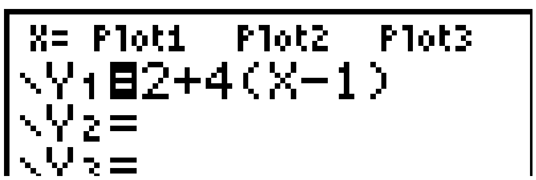
$a_{13} = 50$

Finding Terms Using your Calculator

If you are given a formula for a sequence, you can use the calculator to help you find any term. Let's use the example from before: $a_n = 2 + 4(n - 1)$, find the thirteenth term (a_{13}).

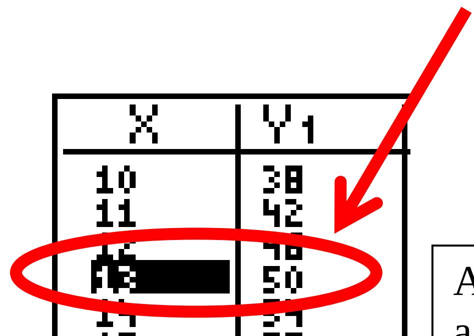
Step 1

Put formula in Y=
Use 'x' in place of 'n'



Step 2

Look in TABLE at x = 13



Answer:
$a_{13} = 50$

Finding a Recursive Formula for an Arithmetic Sequence

Given: the 8th term of an arithmetic sequence is 75, and the 20th term is 39. Find the recursive formula.

Find: a) the first term and common difference
b) the nth term of the sequence

(a) Here's what we find first:

$a_n = a + d(n - 1)$	$a_n = a + d(n - 1)$
$a_8 = a + d(8 - 1)$	$a_{20} = a + d(20 - 1)$
$75 = a + d(8 - 1)$	$39 = a + d(20 - 1)$
$75 = a + 7d$	$39 = a + 19d$

Now, we have a system of two linear equations:

$75 = a + 7d$	Equation 1
$39 = a + 19d$	Equation 2
$36 = -12d$	Equation 1 – Equation 2
$-3 = d$	Common difference

To find the first term…we have to use d and one of the equations!

$a_8 = a + 7d$		$a_{20} = a + 19d$
$75 = a + 7(-3)$	OR	$39 = a + 19(-3)$
$75 = a - 21$		$39 = a - 57$
$96 = a$		$96 = a$

(b) Since we have our first term, and a common difference, we can now use our formula and apply what we know:

$$a_n = a + d(n - 1)$$
$$a_n = 96 + (-3)(n - 1)$$
$$a_n = 99 - 3n$$

Sum of *n* terms of an Arithmetic Sequence

Let $\{a_n\}$ be an arithmetic sequence with first term $a_1 = a$ and **common difference *d***. The sum $\boldsymbol{S_n}$ of the first n terms of $\{a_n\}$ is

$$S_n = \frac{n}{2}\left(2a + d(n-1)\right) = \frac{n}{2}\left(a + a_n\right)$$

Example 2: Find the sum S_n of the first *n* terms of {3n + 5}.

This is asking us to ***find a general formula*** for the sum…

Solution: First few terms → 3(1) + 5 = 8
3(2) + 5 = 11
3(3) + 5 = 14

Common difference: 3 $a_1 = 8$ $a_n \rightarrow 3n + 5$

So, we have the sum 8 + 11 + 14 + … + (3n + 5)

$$S_n = \frac{n}{2}\left(a + a_n\right)$$

(1) Write down formula

$$S_n = \frac{n}{2}\left(8 + (3n + 5)\right)$$

(2) Substitute values

$$S_n = \frac{n}{2}\left(3n + 13\right)$$

(3) This is our solution!

Now, if all we wanted was the sum of the first 20 terms, we only need to plug in…

$$S_n = \frac{n}{2}\left(3n + 13\right)$$

$$S_{20} = \frac{20}{2}\left(3(20) + 13\right)$$

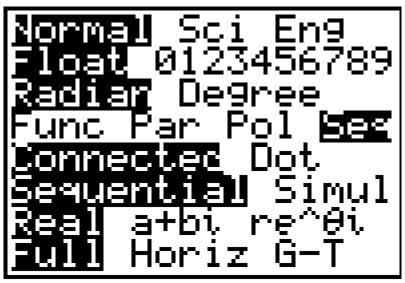
$$S_{20} = 10\left(60 + 13\right) = 730$$

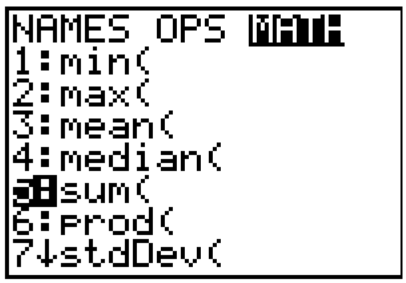
Finding the Sum for an Arithmetic Sequence

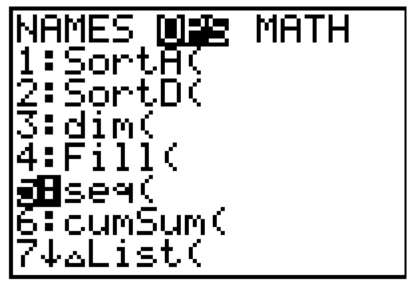
If you are given a formula for a sequence, you can use the calculator to help you find sum for a given number of terms. Let's use the example from before: Find $\boldsymbol{S_{20}}$ for {3n + 5}.

Technique #1: TI 83 or TI84 Family

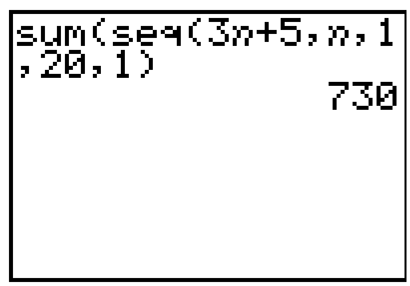
MODE | 2nd LIST | 2nd LIST





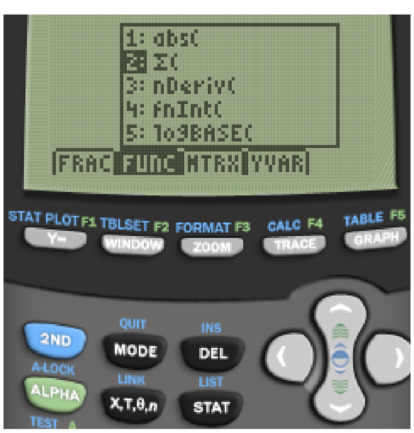


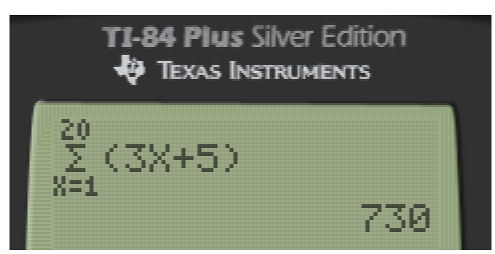
Type in (expression, variable, beginning term, ending term, increment)



Technique #2 for TI84 Family ONLY:

ALPHA WINDOW 2:∑(| Type in information





Thank you TI. You're my new BFF.

Dan Muscarella, 2011

nth Term of a Geometric Sequence

Let $\{a_n\}$ be a geometric sequence with first term $a_1 = a$ and whose **common ratio *r***, the nth term is determined by the formula

$$a_n = ar^{n-1},\ r \neq 0$$

Example 3: Find the ninth term of the geometric sequence

$$10, 9, \frac{81}{10}, \frac{729}{100}, \ldots$$

Solution: Determine the values for a and r…

$a = 10$ and $r = \frac{9}{10}$

$$a_9 = 10\left(\frac{9}{10}\right)^{9-1}$$

$$a_9 = 4.3046721$$

Sum of *n* terms of a Geometric Sequence

Let $\{a_n\}$ be a geometric sequence with first term $a_1 = a$ and **common ratio *r***. The sum S_n of the first n terms of $\{a_n\}$ is

$$S_n = \frac{a(1-r^n)}{1-r},\ r \neq 0,1$$

Example 4: Find the sum S_n of the first n terms of $\left\{\left(\frac{1}{2}\right)^n\right\}$.

Solution: First few terms → $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \cdots$

Common ratio: $\frac{1}{2}$ General term → $\left(\frac{1}{2}\right)^n$

$$S_n = \sum_{k=1}^{n}\left(\frac{1}{2}\right)^k = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \left(\frac{1}{2}\right)^n$$

$$S_n = \sum_{k=1}^{n}\left(\frac{1}{2}\right)^k = \frac{\frac{1}{2}\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}} = \frac{\frac{1}{2}\left[1-\left(\frac{1}{2}\right)^n\right]}{\frac{1}{2}} = 1-\left(\frac{1}{2}\right)^n$$

Sum of an Infinite Geometric Sequence

If $|r| < 1$, the sum of the infinite geometric series is

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}$$

Example 5: Show that the repeating decimal 0.999…= 1

Solution:

$$\mathbf{0.999\ldots = 0.9 + 0.09 + 0.009 + 0.0009 + \ldots}$$

$$0.999\ldots = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \ldots$$

The first term is a = 9/10, and the common ratio is 1/10

$$0.999\ldots = \frac{\frac{9}{10}}{1 - \frac{1}{10}} = \frac{\frac{9}{10}}{\frac{9}{10}} = 1$$

Example 6: Find the sum of the geometric series below:

$$2+\frac{4}{3}+\frac{8}{9}+\ldots$$

Solution: First term is a = 2, and the common ratio is

$$r=\frac{\frac{4}{3}}{2}=\frac{4}{3}\bullet\frac{1}{2}=\frac{2}{3}$$

Be sure to check that $|r|<1$!! Since $|r|=\frac{2}{3}<1$, we can use the formula from above.

$$\sum_{k=1}^{\infty} ar^{k-1}=\frac{a}{1-r}$$

(1) Write down your formula!!

$$\sum_{k=1}^{\infty} 2\left(\frac{2}{3}\right)^{k-1}=\frac{2}{1-\frac{2}{3}}=\frac{2}{\frac{3}{3}-\frac{2}{3}}=\frac{2}{\frac{1}{3}}=6$$

(2) Sub and simplify

$$2+\frac{4}{3}+\frac{8}{9}+\ldots=6$$

(3) Write solution like this…don’t just put ‘6’

You've Got Problems!

Pg. 798 #13, 15, 17, 21 − 41 (eoo)
Pg. 808 #9, 11, 19, 21, 25, 29, 59, 61
Write one formula sheet that contains all of the formulas in these sections.