Administrative:

1. No quarter test - may do quiz
2. Out Friday - notes now/Fri
3. HW 4-3: Pg 256 #21-27 (o), 29-36,
   37-77 (o), 85
4. HW 4-4: Pg 269 #9-63 (eoo), 67-74,
   75-103 (eoo), 111-115 (o), 125
5. Last tests - much better!

Section 4.4 Date_________

Logarithmic Functions

Given a positive base, a logarithm tells you the exponent that is necessary to achieve a certain value. **Logarithmic functions are the *inverses* of exponential functions.**

The **logarithmic function** to the base a, where $a > 0$ and $a \neq 1$, is denoted as $y = \log_a x$.

$$\boxed{y = \log_a x \Leftrightarrow x = a^y}$$

$5^x = y$

$5^x = 25$

$5^x = \frac{1}{25}$

Ans

exp

$y = \log_a x$

base

$a^y = x$

Ex 1 Change from an exponential expression to a logarithmic expression

a) $4.1^3 = m$ $\log_{4.1} m = 3$	b) $e^m = 8$ $\ln_e 8 = m$	c) $w^5 = 32$ $\log_w 32 = 5$

Ex 2 Change from a logarithmic expression to an exponential expression

a) $\log_a 3 = 7$ $a^7 = 3$	b) $\log_e b = -2$ $e^{-2} = b$	c) $\log_2 9 = c$ $2^c = 9$

Ex 3 Find the exact value of each expression.

a) $\log_2 32 = h$	b) $\log_3 \frac{1}{81} = a$	c) $\log_{\frac{1}{2}} 8 = i$	d) $\log_{125} 5 = l$

a)
$$2^h = 32$$
$$2^h = 2^5$$
$$\boxed{h = 5}$$

b)
$$3^a = \frac{1}{81}$$
$$3^a = 3^{-4}$$
$$\boxed{a = -4}$$

c)
$$\frac{1}{2}^i = 8$$
$$(2^{-1})^i = 2^3$$
$$2^{-i} = 2^3$$
$$-i = 3$$
$$\boxed{i = -3}$$

d)
$$125^l = 5$$
$$5^{3l} = 5^1$$
$$3l = 1$$
$$\boxed{l = 1/3}$$

Domain of a Logarithmic Function: Logarithmic functions are the ___inverses___ of exponential functions, so given that $y = \log_a x$ is defined as $x = a^y$...

- Domain $_{\text{logarithmic function}}$ = Range $_{\text{exponential function}}$ i.e. ___$(0, \infty)$___
- Range $_{\text{logarithmic function}}$ = Domain $_{\text{exponential function}}$ i.e. ___$(-\infty, \infty)$___

$y = \log_a x$ (defining equation: $x = a^y$)	Domain: $0 < x < \infty$	Range: $-\infty < y < \infty$

★ Note: The argument of the logarithmic function must be ___positive___.

$$\log_a(\text{ARGUMENT})$$

Properties of the Graph of a Logarithmic Function:

1. Domain: $(0, \infty)$

   Range: $(-\infty, \infty)$
2. x-intercept: $(1, 0)$ y-intercept: None
3. Asymptote: $x = 0$ (vertical)
4. The log function is ***decreasing*** if $0 < a < 1$ and ***increasing*** if $a > 1$.
5. The graph contains the points $(a, 1)$, $(1, 0)$ and $(\frac{1}{a}, -1)$.
6. The graph of the log function is smooth and continuous with no corners or gaps.

Natural Logarithm Function: a log function that is base e. $y = \log_e x = \ln x \Leftrightarrow x = e^y$

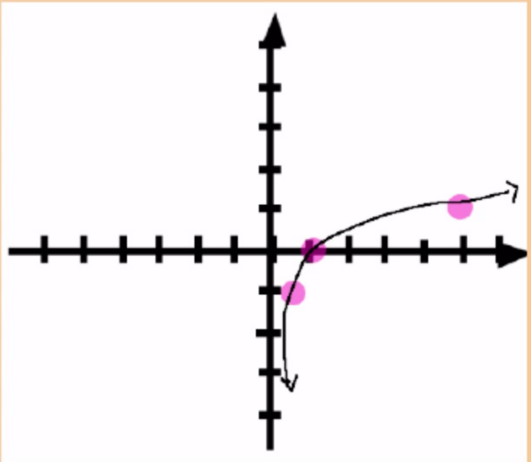
Common Logarithm Function: a log function that is base 10 $y = \log_{10} x = \log x \Leftrightarrow x = 10^y$

Graph the logarithmic function.

$$f(x) = \log_5 x$$

a = 5

$$\left(a, 1\right) \qquad \left(5, 1\right)$$
$$\left(1, 0\right) \qquad \left(1, 0\right)$$
$$\left(\tfrac{1}{a}, -1\right) \qquad \left(\tfrac{1}{5}, -1\right)$$



Vertical Asymptote (VA): x = 0
Domain: (0, ∞)
Range: (-∞, ∞)

Alternative Way using inverse functions

Graph the logarithmic function.

$$f(x) = \log_5 x$$

$$y = \log_5 x$$

$$5^y = x$$

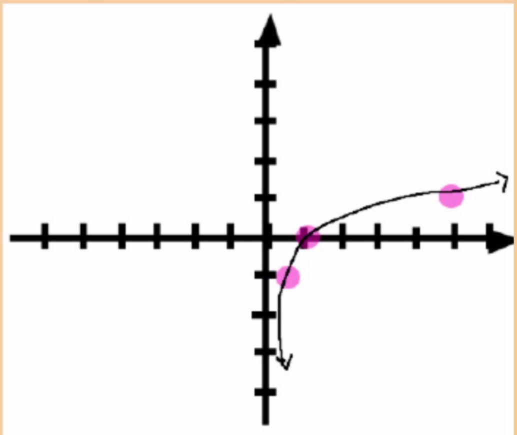
$5^x = y$

x	y
-1	1/5
0	1
1	5

$\Longrightarrow$

$x = 5^y$

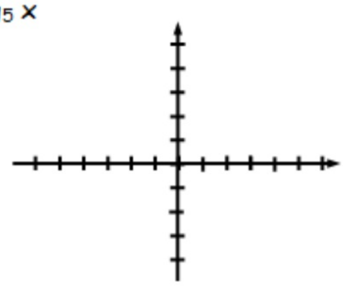
x	y
1/5	-1
1	0
5	1



Vertical Asymptote (VA): x = 0
Domain: (0, ∞)
Range: (-∞, ∞)

Ex 5 Graph.

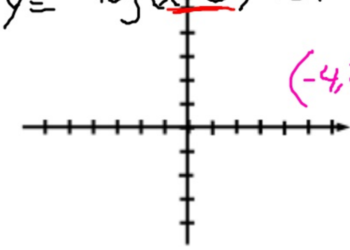
a) $y = \log_5 x$

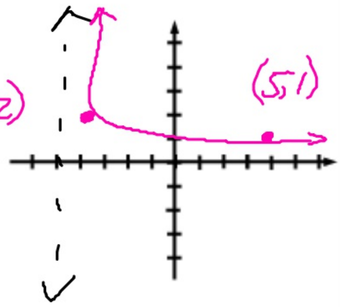


Domain:_________ Range__________

b) $y = 2 - \log(x + 5)$

$y = -\log(x+5) + 2$





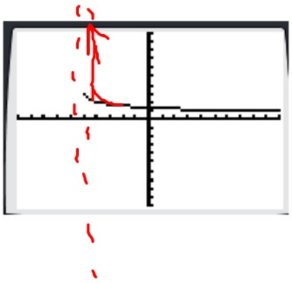
Domain:____________ Range______________

X See VIDEO

$x + 5 > 0$

$x > -5$

VA : $x = -5$



a,

Domain: (-5, ∞)
Range: (-∞, ∞)

Find the domain of the logarithmic function.

$$f(x) = \log_4(\underline{x + 3})$$ ARGUMENT > 0

$x + 3 > 0$

$x > -3$

$(-3, \infty)$

Find the domain of the logarithmic function.

$$g(x) = \log_3 x^2$$

$$x^2 > 0$$

$$x < 0 \qquad x > 0$$

$$(-\infty, 0) \cup (0, \infty)$$

Find the domain of the logarithmic function.

$$h(x) = \log_{\frac{1}{2}}\left(\frac{x+2}{x-2}\right)$$

$x + 2 > 0$ $\qquad\qquad$ $x - 2 \neq 0$

$x > -2$ $\qquad\qquad$ $x \neq 2$

Test:

$+ \quad -2 \quad - \quad 2 \quad +$

$x = -3$

$\frac{-3+2}{-3-2} = \frac{-1}{-5}$

$+$

$x = 0$

$\frac{2}{-2}$

$-$

$x = 3$

$\frac{3+2}{3-2} \Rightarrow +$

Domain: $(-\infty, -2) \cup (2, \infty)$

Ex 6 Solve. Discard any extraneous solutions.

a) $\log_6 36 = 5x + 3$

$$6^{5x+3} = 36$$

$$6^{5x+3} = 6^2$$

$$5x + 3 = 2$$

$$5x = -1$$

$$\boxed{x = -1/5}$$

b) $\ln e^{-2x} = 8$

$$-2x = 8$$

$$\boxed{x = -4}$$

Check to see if extraneous

c) $\log_5 (x^2 + 7x + 7) = 2$

$$5^2 = x^2 + 7x + 7$$

$$25 = x^2 + 7x + 7$$

$$0 = x^2 + 7x - 18$$

$$0 = (x+9)(x-2)$$

$$x + 9 = 0 \qquad x - 2 = 0$$

$$\boxed{x = -9} \qquad \boxed{x = 2}$$

d) $e^{2x+5} = 8$

$$\ln e^{2x+5} = \ln 8$$

$$2x + 5 = \ln 8$$

$$2x = \ln 8 - 5$$

$$x = \frac{\ln 8 - 5}{2}$$

$$x \approx -1.46$$