Properties of Logarithms

Ex 1 Prove the following theorems.

a) $\log_a 1 = 0$ $a^0 = 1$ $1 = 1$	b) $\log_a a = 1$ $a^1 = a$ $a = a$ ✓
c) $a^{\log_a M} = M$	d) $\log_a a^r = r$ $a^r = a^r$

See Text

$\log_3 3 = \underline{x}$

$3^x = 3$

$x = 1$

Ex 2 Prove the following properties. In these properties, M, N, and a are positive real numbers with $a \neq 1$, and r is a real number.

a) $\log_a (MN) = \log_a M + \log_a N$	b) $\log_a M^r = r \log_a M$
$\log_a(5x) =$ $\log_a 5 + \log_a x$	ex $\log x^2$ $2 \log x$

ex $\log \sqrt{x} = \log x^{\frac{1}{2}}$

$= \frac{1}{2} \log x$

Summary of the Properties of Logarithms: In these properties, M, N, and a are positive real numbers with $a \neq 1$, and r is a real number.

Special Values: $\log_a 1 = 0$ and $\log_a a = 1$
Inverse Properties: $a^{\log_a M} = M$ and $\log_a a^r = r$
The log of a product equals the sum of the logs, i.e. $\log_a (MN) = \log_a M + \log_a N$.
The log of a quotient equals the difference of the logs, i.e. $\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$.
The log of a power equals the product of the power and the log, i.e. $\log_a M^r = r \log_a M$.
If M = N, then $\log_a M = \log_a N$.
If $\log_a M = \log_a N$, then M = N.

$\log_a a = 1$ $\quad$ $\log_a 1 = 0$ $\quad$ MADS

$a^1 = a$ $\quad$ $a^0 = 1$

$a = a$

Change-of-Base Formula: a, b, and M are positive real numbers, and a ≠ 1, b ≠ 1.

In general: $\log_a M = \dfrac{\log_b M}{\log_b a}$ **For calculators**: $\log_a M = \dfrac{\log M}{\log a} = \dfrac{\ln M}{\ln a}$

Ex 3 Use properties of logarithms to find each expression's exact value. (No calculators!)

a) $\ln e^{\sqrt{2}}$

$\ln_e e$

$\sqrt{2}$

b) $4^{\log_4 9}$

9

$5^a \cdot 5^b$

5^{a+b}

c) $\log_3 8 \cdot \log_8 9 = x$

$$\frac{\log 8}{\log 3} \cdot \frac{\log 9}{\log 8} = x$$

$$\frac{\log 9}{\log 3} = x$$

$$\log_3 9 = x$$

$$3^x = 9$$

$$3^x = 3^2$$

$$\boxed{x = 2}$$

d) $5^{\overset{a}{\log_5 6} + \overset{b}{\log_5 7}}$

$$5^{\log_5 6} \cdot 5^{\log_5 7}$$

$$6 \cdot 7$$

$$\boxed{42}$$

$$7^3 \cdot 7^{2x}$$

$$7^{3+2x}$$

Ex 4 Given ln 2 = a and ln 3 = b, write each logarithm in terms of a and b.

a) $\ln\frac{2}{3}$	b) $\ln 0.5 = \ln\frac{1}{2}$	c) $\ln\sqrt[4]{\frac{2}{3}}$

a)

$\ln 2 - \ln 3$

$\boxed{a - b}$

b)

$\ln 1 - \ln 2$

$\ln 1 - a$

$0 - a$

$\boxed{-a}$

OR

$\ln\frac{1}{2} = \ln 2^{-1}$

$= -\ln 2$

$= -a$

c)

$\ln\left(\frac{2}{3}\right)^{\frac{1}{4}}$

$\frac{1}{4}\ln\left(\frac{2}{3}\right)$

$\frac{1}{4}\left[\ln 2 - \ln 3\right]$

$\frac{1}{4}\left[a - b\right]$

$\frac{1}{4}a - \frac{1}{4}b$

$\frac{a-b}{4}$

<u>Ex 5</u> Expand each logarithm. (Write each expression as a sum and/or difference of logs.)

a) $\ln \frac{x}{e^x}$	b) $\log\left[\frac{x^3\sqrt{x+1}}{(x-2)^2}\right], x>2$

$\ln x - \ln e^x$

$\boxed{\ln x - x}$

$\log x^3(x+1)^{1/2} - \log(x-2)^2, x>2$

$\log x^3 + \log(x+1)^{1/2} - 2\log(x-2), x>2$

$\boxed{3\log x + \frac{1}{2}\log(x+1) - 2\log(x-2), x>2}$

Ex 6 Condense each logarithm. (Wri

a) $\log\left(\dfrac{x^2+2x-3}{x^2-4}\right)-\log\left(\dfrac{x^2+7x+6}{x+2}\right)$

$$\log\left(\frac{(x+3)(x-1)}{(x+2)(x-2)}\right)-\log\left(\frac{(x+6)(x+1)}{(x+2)}\right)$$

$$\log\left(\frac{\frac{(x+3)(x-1)}{(x+2)(x-2)}}{\frac{(x+6)(x+1)}{(x+2)}}\right)$$

$$\log\left(\frac{(x+3)(x-1)}{\cancel{(x+2)}(x-2)}\cdot\frac{\cancel{(x+2)}}{(x+6)(x+1)}\right)$$

$$\log\left(\frac{x^2+2x-3}{(x-2)(x^2+7x+6)}\right)$$

Ex 6 Condense each logarithm. (Wri

a) $\log\left(\frac{x^2+2x-3}{x^2-4}\right)-\log\left(\frac{x^2+7x+6}{x+2}\right)$

$$\log\left(\frac{\left(\frac{x+2x-3}{x^2-4}\right)}{\left(\frac{x^2+7x+6}{x+2}\right)}\right)$$

$$\log\left(\frac{x^2+2x-3}{x^2-4}\cdot\frac{x+2}{x^2+7x+6}\right)$$

b) $3 \log_5 (3x+1) - 2 \log_5 (2x-1) - \log_5 x$

$$\log_5 (3x+1)^3 - \log_5 (2x-1)^2 - \log_5 x$$

$$\log_5 \left(\frac{(3x+1)^3}{(2x-1)^2} \right) - \log_5 x$$

$$\log_5 \left(\frac{\frac{(3x+1)^3}{(2x-1)^2}}{x} \right) \qquad \log_5 \left(\frac{1}{x} \right) \left(\frac{(3x+1)^3}{(2x-1)^2} \right)$$

$$\log_5 \left(\frac{(3x+1)^3}{x(2x-1)^2} \right)$$

b) $3 \log_5 (3x+1) - 2 \log_5 (2x - 1) - \log_5 x$

$$\log_5(3x+1)^3 - \log_5(2x-1)^2 - \log_5 x$$

$$\log_5\left(\frac{(3x+1)^3}{(2x-1)^2}\right) \cancel{-\log_5 x}$$

$$\log_5\left(\frac{\frac{(3x+1)^3}{(2x-1)^2}}{x}\right)$$

$$\log_5\left(\frac{(3x+1)^3}{x(2x-1)^2}\right)$$

$$\log_5 (3x+1)^3 - \left(\log_5 (2x-1)^2 + \log_5 x\right)$$

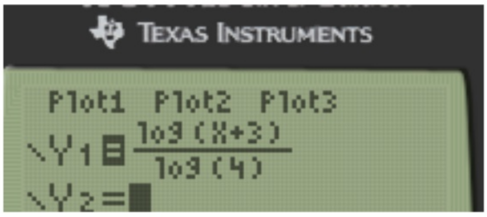
$$\log_5 (3x+1)^3 - \left(x(2x-1)^2\right)$$

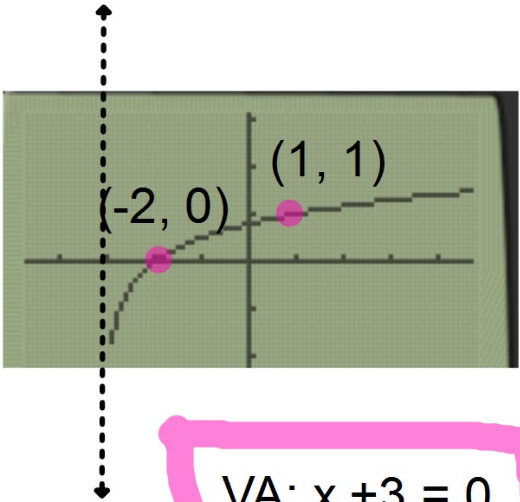
$$\log_5 \frac{(3x+1)^3}{x(2x-1)^2}$$

Ex 7 Use the Change-of-Base Formula to do the following:

a) Evaluate $\log_{\frac{1}{2}} 15$. $$\frac{\log 15}{\log \frac{1}{2}} = \frac{\log 15}{\log 1 - \log 2} = \frac{\log 15}{-\log 2}$$	b) Evaluate $\log_{\pi} \sqrt{2}$. $$\frac{\log \sqrt{2}}{\log n} = \frac{\log (2)^{1/2}}{\log n} = \frac{\frac{1}{2}\log (2)}{\log n} = \frac{\log(2)}{2\log n}$$

c) Graph $y = \log_4(x + 3)$.





$$\boxed{\begin{aligned} \text{VA: } x + 3 &= 0 \\ x &= -3 \end{aligned}}$$

HW: Quiz Review

Quiz: 14 problems
All Non-Calc
4.3-4.5