30, 60, 90, 120, 150, 180, ...

60, 120, 180, ...

45, 90, 135, 180, 225, ...

15°

60 - 45

45 - 30

135 - 120

Double-Angle and Half-Angle Formulas:

Function	**Double-Angle Formulas**	**Half-Angle Formulas:** The + or - sign is determined by the quadrant of $\frac{\alpha}{2}$.
Cosine	$\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $\cos(2\theta) = 1 - 2\sin^2\theta$ $\cos(2\theta) = 2\cos^2\theta - 1$	$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$
Sine	$\sin(2\theta) = 2\sin\theta\cos\theta$	$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$
Tangent	$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$	$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$

Is the value of each expression below *positive* or *negative*? (Consider the angle's quadrant!)

Example A) $\cos 102.5°$ $\alpha \to \frac{\alpha}{2}$	1. $\sin 85°$ pos	2. $\tan 207.5°$ pos	3. $\cos 283.5°$ pos
Example B) $\tan\frac{\alpha}{2}$ if $\sin\alpha = \frac{7}{12}$, $\boxed{\frac{\pi}{2} < \alpha < \pi}$ $\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$ pos.	4. $\cos\frac{\alpha}{2}$ if $\sin\alpha = \frac{9}{10}$, $\frac{\pi}{2} < \alpha < \pi$ QUAD I $\rightarrow$ $\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$ pos.	5. $\tan\frac{\alpha}{2}$ if $\cos\alpha = \frac{7}{12}$, $\frac{\pi}{4} < \alpha < \frac{\pi}{2}$ $\frac{\pi}{8} < \frac{\alpha}{2} < \frac{\pi}{4}$	6. $\sin\frac{\alpha}{2}$ if $\tan\alpha = \frac{25}{12}$, $\pi < \alpha < \frac{3\pi}{2}$ Q II $\frac{\pi}{2} < \frac{\alpha}{2} < \frac{3\pi}{4}$ sin is pos.

Rewrite the expression so that one could use the Half-Angle formula. Will the value of the expression be *positive* or *negative*? (Don't evaluate!)

Example C) $\sin 35°$ $\sin\left(\frac{70}{2}\right)$	7. $\tan 165°$ $\tan\left(\frac{330}{2}\right)$	8. $\cos 67.5°$ $\cos\left(\frac{135}{2}\right)$
Example D) $\cos\frac{9\pi}{8}$ $\cos\left(\frac{18\pi}{8}\right)$	9. $\sin\frac{17\pi}{8}$ $\sin\left(\frac{34\pi}{8}\right)$	10. $\tan\frac{\pi}{12}$ $\tan\left(\frac{2\pi}{12}\right)$

reduce

$\cos\left(\frac{9\pi}{4}\right)$ $\qquad$ $\sin\frac{17\pi}{4}$ $\qquad$ $\tan\left(\frac{\pi}{6}\right)$

coterminal

$\cos(\pi/4)$ $\qquad$ $\sin\left(\frac{\pi}{4}\right)$ $\qquad$ positive

($\frac{\pi}{6}$ is 2x's $\frac{\pi}{12}$)

positive $\qquad$ pos.

$$\frac{9\pi}{8} = \frac{x}{2}$$

$$\frac{18\pi}{8} = x$$

$$\frac{17\pi}{8} = \frac{x}{2}$$

$$\frac{34\pi}{8} = x$$

$$\frac{\pi}{12} = \frac{x}{2}$$

$$\frac{2\pi}{12} = x$$

$$\frac{\pi}{6} = x$$

Find the exact value of each of the following using the Half-Angle formula.

Example E) $\sin 165°$ $= \sin\left(\frac{330}{2}\right)$ $\sin 165 = \sqrt{\frac{1-\cos 330}{2}}$	11. $\cos 105°$
Example F) $\tan\frac{5\pi}{8}$	12. $\tan\frac{3\pi}{8}$

Example E) $\sin 165° = \sin\left(\frac{330}{2}\right)$

$$\sin 165 = \sqrt{\frac{1-\cos 330}{2}}$$

$$\sin 165 = \sqrt{\left(\frac{1}{2}\right)\left(1-\cos 330\right)}$$

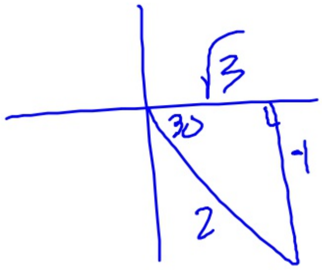
$$= \sqrt{\frac{1}{2}\left(1-\frac{\sqrt{3}}{2}\right)}$$

$$= \sqrt{\frac{1}{2}\left(\frac{2}{2}-\frac{\sqrt{3}}{2}\right)}$$

$$= \sqrt{\frac{1}{2}\left(\frac{2-\sqrt{3}}{2}\right)}$$

$$= \sqrt{\frac{2-\sqrt{3}}{4}}$$

$$= \frac{\sqrt{2-\sqrt{3}}}{2}$$



$$\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$

$$\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}$$

$$\cos 105 = \cos\left(\frac{210}{2}\right)$$

$$\cos 105 = -\sqrt{\left(\frac{1}{2}\right)\left(1+\cos 210\right)}$$

$$= -\sqrt{\left(\frac{1}{2}\right)\left(1-\frac{\sqrt{3}}{2}\right)}$$

$$= -\sqrt{\left(\frac{1}{2}\right)\left(\frac{2}{2}-\frac{\sqrt{3}}{2}\right)}$$

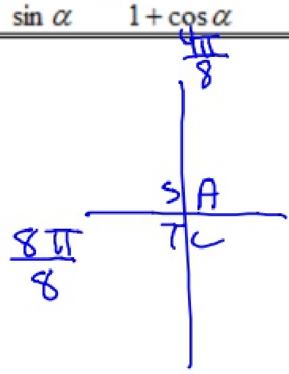
$$= -\sqrt{\left(\frac{1}{2}\right)\left(\frac{2-\sqrt{3}}{2}\right)}$$

$$= -\sqrt{\frac{2-\sqrt{3}}{4}}$$

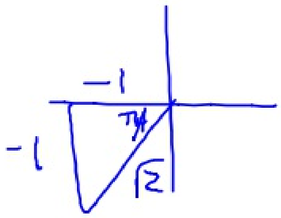
$$= \frac{-\sqrt{2-\sqrt{3}}}{2}$$

$\tan\frac{5\pi}{8} = \tan\frac{\frac{5\pi}{4}}{2}$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$$



$\frac{\sin\frac{5\pi}{4}}{1+\cos\frac{5\pi}{4}}$



$\frac{1-\cos\frac{5\pi}{4}}{\sin\frac{5\pi}{4}}$

$\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{-\frac{\sqrt{2}}{2}}$

$\frac{1+\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$

$\frac{\frac{2+\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$

$\frac{2+\sqrt{2}}{2}\cdot\left(\frac{2}{-\sqrt{2}}\right)$

$\frac{2+\sqrt{2}}{-\sqrt{2}}$

$\frac{2+\sqrt{2}}{-\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$\frac{2\sqrt{2}+2}{-2} = \frac{2\sqrt{2}}{-2} + \frac{2}{-2}$

$\boxed{= -\sqrt{2}-1} = -1(\sqrt{2}+1)$

Find the exact value of each of the following using the Double-Angle formula.

Example G) $\tan\left[2\cos^{-1}\left(-\frac{\sqrt{3}}{4}\right)\right]$	13. $\cos\left(2\tan^{-1}\frac{\sqrt{11}}{5}\right)$
14. If $\csc\theta = -\sqrt{5}$ and $\cos\theta < 0$, what is $\sin(2\theta)$?	15. If $\tan\theta = -3$ and $0 < \theta < \pi$, what is $\cos(2\theta)$?

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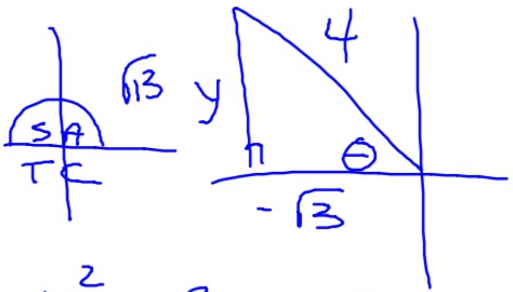
19–27 (o)

Example G) $\tan\left[2\cos^{-1}\left(-\frac{\sqrt{3}}{4}\right)\right] \rightarrow \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\dfrac{\sqrt{13}}{\sqrt{3}} = \sqrt{\dfrac{13}{3}}$

$\tan[2\theta]$



$$= \frac{2\left(-\frac{\sqrt{13}}{\sqrt{3}}\right)}{1-\left(-\frac{\sqrt{13}}{\sqrt{3}}\right)^2}$$

$4^2 = y^2 + (-\sqrt{3})^2$

$16 = y^2 + 3$

$13 = y^2$

$\sqrt{13} = y$

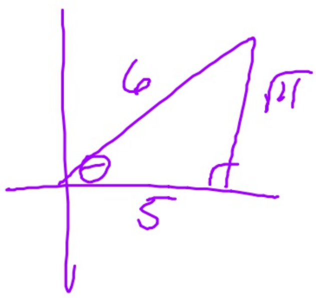
$$= \frac{2\left(-\frac{\sqrt{39}}{3}\right)}{1-\frac{13}{3}}$$

$$= \frac{-\frac{2\sqrt{39}}{3}}{\frac{3}{3}-\frac{13}{3}}$$

$$= \frac{-\frac{2\sqrt{39}}{3}}{\frac{-10}{3}} = -\frac{2\sqrt{39}}{3} \cdot -\frac{3}{10} = \boxed{\frac{\sqrt{39}}{5}}$$

ng using the Double-

13. $\cos\left(\underbrace{2\tan^{-1}\frac{\sqrt{11}}{5}}\right) \rightarrow \cos(2\theta) \rightarrow 1-2\sin^2\theta$



$r^2 = 11+25$

$r^2 = 36$

$r = 6$

$$= 1-2\left(\frac{\sqrt{11}}{6}\right)^2$$

$$= 1-2\left(\frac{11}{36}\right)$$

$$= \frac{36}{36}-\frac{22}{36}$$

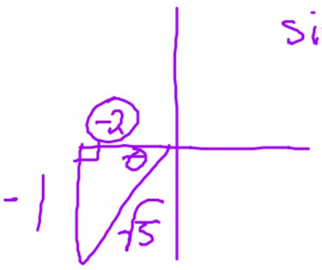
$$= \frac{14}{36}$$

$$= \frac{7}{18}$$

14. If $\csc\theta = -\sqrt{5}$ **and** $\cos\theta < 0$**, what is** $\sin(2\theta)$**?**	**15. If** $\tan\theta = -3$ **and** $0 < \theta < \pi$**, what is** $\cos(2\theta)$**?**

$$\frac{1}{\sin\theta} = -\sqrt{5}$$

$$\sin\theta = -\frac{1}{\sqrt{5}}$$



$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\left(-\frac{1}{\sqrt{5}}\right)\left(\frac{-2}{\sqrt{5}}\right)$$

$$= \frac{4}{5}$$

$$x^2 + (-1)^2 = \left(\sqrt{5}\right)^2$$

$$x^2 + 1 = 5$$

$$x^2 = 4$$

$$x = \pm\sqrt{4}$$

$$\boxed{x = -2}$$

Sum and Difference Formulas:

Function	Sum Formula	Difference Formula
Cosine	$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$	$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
Sine	$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$	$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$
Tangent	$\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$	$\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

Use the sum or difference of special angle values to rewrite the angle measures below.

Example A) 105° 45+60 135−30	1. 255° 225+30 300−45 210+45	2. 195°
Example B) $\frac{13\pi}{12}$	3. $\frac{17\pi}{12}$	4. $\frac{5\pi}{12}$

30, 60, 90, 120, 150, 180, 210, ...

45, 90, 135, 180, 225, 270, . . .

60, 120, 180, 240, 300, 360

30 $\frac{\pi}{6}$ $\frac{2\pi}{12}$ $\frac{4\pi}{12}$ $\frac{6\pi}{12}$ $\frac{8\pi}{12}$

45 $\frac{\pi}{4}$ $\frac{3\pi}{12}$ $\frac{6\pi}{12}$ $\frac{9\pi}{12}$ $\frac{12\pi}{12}$ $\frac{15\pi}{12}$

60 $\frac{\pi}{3}$ $\frac{2\pi}{3}$, $\frac{3\pi}{3}$, $\frac{4\pi}{3}$, $\frac{5\pi}{3}$

$\frac{4\pi}{12}$ $\frac{8\pi}{12}$ $\frac{12\pi}{12}$ $\frac{16\pi}{12}$ $\frac{20\pi}{12}$

$\boxed{\frac{13\pi}{12}}$ $\frac{16\pi}{12} - \frac{3\pi}{12}$

$\boxed{\frac{17\pi}{12}}$ $\frac{15\pi}{12} + \frac{2\pi}{12}$ $\frac{20\pi}{12} - \frac{3\pi}{12}$

$\frac{9\pi}{12} + \frac{8\pi}{12}$

4 4

$$\frac{\sqrt{2}+\sqrt{6}}{4}$$

$\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$ X

$\frac{1}{4}\left(\sqrt{2}+\sqrt{6}\right)$

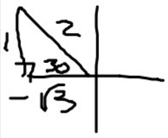
$\frac{\sqrt{2}}{4}\left(1+\sqrt{3}\right)$ $\frac{\sqrt{2}\left(1+\sqrt{3}\right)}{4}$

Find the exact value of each trigonometric function.

Example C) $\sin 195°$	5. $\cos 165°$

$$\sin(150+45) = \sin 150 \cos 45 + \cos 150 \sin 45$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$



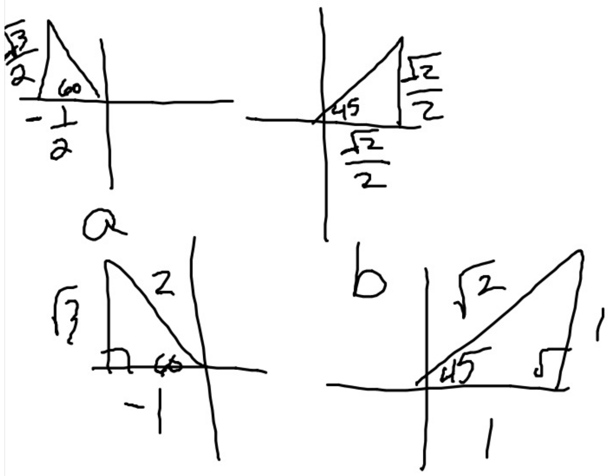
$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2}-\sqrt{6}}{4}$$

Example D) $\tan\frac{11\pi}{12}$	6. $\tan\frac{19\pi}{12}$

$$\frac{8\pi}{12}+\frac{3\pi}{12}$$

$$\frac{2\pi}{3}+\frac{\pi}{4}$$



$$\tan(a+b)=\frac{\tan a+\tan b}{1-\tan a\tan b}$$

$$\frac{-\sqrt{3}+1}{1-(-\sqrt{3})(1)}$$

$$\boxed{\frac{1-\sqrt{3}}{1+\sqrt{3}}}$$

HW: complete rest of problems in these notes

See ex 4 from Notes 6.4!

Example: Given $\sin\alpha = -\frac{3}{5}, \frac{3\pi}{2} < \alpha < 2\pi$ & $\tan\beta = -\frac{\sqrt{5}}{2}, \frac{\pi}{2} < \beta < \pi$, find the exact value of ...

E) $\sin(\alpha + \beta)$	F) $\tan(\alpha - \beta)$

TRY: Given $\cos\alpha = -\frac{12}{13}$, $\frac{\pi}{2} < \alpha < \pi$ and $\sin\beta = -\frac{\sqrt{7}}{4}$, $-\frac{\pi}{2} < \beta < 0$, find the exact value of ...

7. $\cos(\alpha - \beta)$	8. $\tan(\alpha + \beta)$

Find the exact value of each expression.

Example G) $\cos\left[\tan^{-1}\frac{5}{12} - \sin^{-1}\left(-\frac{3}{5}\right)\right]$	9. $\cos\left[\tan^{-1}\frac{4}{3} + \cos^{-1}\frac{12}{13}\right]$