EX 1] Find the exact value of each expression: $\sin\left(\frac{3\pi}{4}+\frac{5\pi}{6}\right)$

a) $\sin(u+v) = \sin u\cos v + \cos u\sin v$ (1) write formula

$\sin\left(\frac{3\pi}{4}+\frac{5\pi}{6}\right) = \sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{5\pi}{6}\right) + \cos\left(\frac{3\pi}{4}\right)\sin\left(\frac{5\pi}{6}\right)$ (2) Substitute in formula

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{-\sqrt{3}}{2}\right) + \left(\frac{-\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$ (3) Evaluate trig functions

$= \left(\frac{-\sqrt{6}}{4}\right) + \left(\frac{-\sqrt{2}}{4}\right)$ (4) Simplify

$= \frac{-\sqrt{6}-\sqrt{2}}{4}$ (5) Simplify

$= -\frac{\sqrt{6}+\sqrt{2}}{4}$ (6) Another way to write

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b) $\sin\left(\frac{3\pi}{4}\right) + \sin\left(\frac{5\pi}{6}\right)$

Since these angles are on the unit circle, just evaluate & simplify!!!

$\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right) = \frac{\sqrt{2}+1}{2}$

EX 5] Verify the identity:

$\cos(x+y) + \cos(x-y) = 2\cos x\cos y$

$(\cos x\cos y - \sin x\sin y) + (\cos x\cos y + \sin x\sin y)$ (1) Apply formulas

$\cos x\cos y - \sin x\sin y + \cos x\cos y + \sin x\sin y$ (2) Simplify

$2\cos x\cos y$ (3) Simplify

EX 2] Use the difference identities to find the exact values of the sin, cos, and tan of the angle

a) $15^\circ = 45^\circ - 30^\circ$ This tells us that we want to make u = 45° and v = 30°
We choose 45° and 30° since both numbers in that combination are on the unit circle. We could have chosen u = 60 and v = 45, or u = 135 and v = 120, but we didn't, so let's move on…

SIN

$\sin(u-v) = \sin u\cos v - \cos u\sin v$ (1) Write formula

$\sin(45°-30°) = \sin(45°)\cos(30°) - \cos(45°)\sin(30°)$ (2) Substitute in formula

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$ (3) Evaluate trig functions

$= \left(\frac{\sqrt{6}}{4}\right) - \left(\frac{\sqrt{2}}{4}\right)$ (4) Simplify

$= \frac{\sqrt{6}-\sqrt{2}}{4}$ (5) Simplify

$= \frac{\sqrt{2}}{4}\left(\sqrt{3}-1\right)$ *(6) Another way to write

Look back at Step (3) and factor out $\frac{\sqrt{2}}{4}$

COS

$\cos(u-v) = \cos u\cos v + \sin u\sin v$ (1) write formula

$\cos(45°-30°) = \cos(45°)\cos(30°) + \sin(45°)\sin(30°)$ (2) Substitute in formula

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$ (3) Evaluate trig functions

$= \left(\frac{\sqrt{6}}{4}\right) + \left(\frac{\sqrt{2}}{4}\right)$ (4) Simplify

$= \frac{\sqrt{6}+\sqrt{2}}{4}$ (5) Simplify

$= \frac{\sqrt{2}}{4}\left(\sqrt{3}+1\right)$ *(6) Another way to write

TAN

Tan(u-v) $= \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$ (1) write formula

Tan(45°–30°) $= \dfrac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$ (2) Substitute in formula

$= \dfrac{1 - \left(\dfrac{\sqrt{3}}{3}\right)}{1 + (1)\left(\dfrac{\sqrt{3}}{3}\right)}$ (3) Evaluate trig functions

$= \dfrac{\dfrac{3}{3} - \left(\dfrac{\sqrt{3}}{3}\right)}{\dfrac{3}{3} + \left(\dfrac{\sqrt{3}}{3}\right)}$ (4) Get LCD in numerator & denominator

$= \dfrac{3 - \sqrt{3}}{3} \div \dfrac{3 + \sqrt{3}}{3}$ (5) Rewrite & simplify step (4)

$= \dfrac{3 - \sqrt{3}}{3} \bullet \dfrac{3}{3 + \sqrt{3}}$ (6)

$= \dfrac{3 - \sqrt{3}}{\cancel{3}} \bullet \dfrac{\cancel{3}}{3 + \sqrt{3}}$ (7) Cancel

$= \dfrac{3 - \sqrt{3}}{3 + \sqrt{3}}$ (8) Simplify

$= \dfrac{3 - \sqrt{3}}{3 + \sqrt{3}} \bullet \dfrac{3 - \sqrt{3}}{3 - \sqrt{3}}$ (9) Multiply by conjugate of denominator

$= \dfrac{9 - 6\sqrt{3} + 3}{9 - 3}$ (10) Make sure you FOIL correctly!

$= \dfrac{12 - 6\sqrt{3}}{6}$ (11) Simplify

$= 2 - \sqrt{3}$ (12) Simplify

Answers for b) $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$

$\text{Sin}\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$ $\text{Cos}\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$ $\text{Tan}\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = -2-\sqrt{3}$

EX 3] Use the sum or difference identities to write the expression as the sin, cos, or tan of the angle.

$\frac{\tan 140^\circ - \tan 60^\circ}{1+\tan 140^\circ \tan 60^\circ}$ → u = 140° v = 60°

Is this the *sum formula* for tan or the *difference formula*?

ANSWER: The difference formula….

So, we have $\tan(u - v) = \tan(140° - 60°)$

$= \tan 80°$

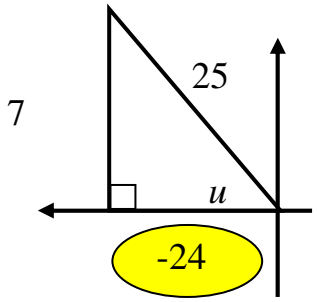
EX 4] Given: $\sin u = \frac{7}{25}$ $\frac{\pi}{2} < u < \pi$, QUAD II

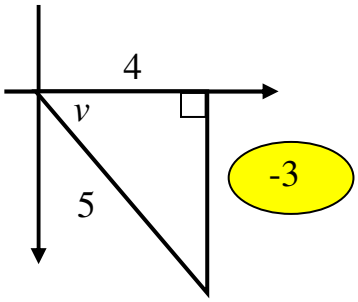
$\cos v = \frac{4}{5}$ $\frac{3\pi}{2} < v < 2\pi$ QUAD IV

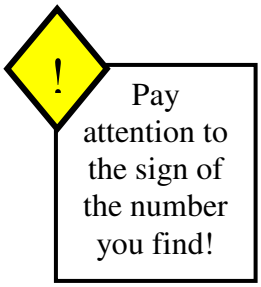
Find the exact value of the trigonometric function: $\sin(u+v)$

Solution:

(1) Draw picture for **each situation** & find missing side by using *Pythagorean Theorem*







$\sin(u+v) = \sin u\cos v + \cos u\sin v$ (2) Write formula

$= \left(\frac{7}{25}\right)\left(\frac{4}{5}\right) + \left(\frac{-24}{25}\right)\left(\frac{-3}{5}\right)$ (3) Substitute in formula

$= \frac{28}{125} + \frac{72}{125}$ (4) Simplify

$= \frac{100}{125}$ (5) Simplify

$= \frac{4}{5}$ (6) Simplify

EX6] Find all the solutions of the equation in the interval $[0, 2\pi)$

$\tan(x + \pi) + 2\sin(x+\pi) = 0$

$\frac{\tan x+\tan\pi}{1-\tan x\tan\pi} + 2(\sin x\cos\pi + \cos x\sin\pi) = 0$ (1) Apply formulas

$\frac{\tan x+0}{1-\tan x(0)} + 2(\sin x(-1)+ \cos x(0)) = 0$ (2) Evaluate

$\frac{\tan x}{1} + 2(-\sin x + 0) = 0$ (3) Simplify

! Don't solve by moving the $2\sin x\cos x$ to the other side and dividing by $\sin x$

$\frac{\sin x}{\cos x} - 2\sin x = 0$ (4) Simplify

$\sin x - 2\sin x\cos x = 0$ (5) Multiply by $\cos x$

$\sin x(1 - 2\cos x) = 0$ (6) Factor out $\sin x$

$\sin x = 0$ $\quad$ $1 - 2\cos x = 0$ (7) Set each factor equal to 0 and solve

$x = 0, \pi$ $\quad$ $\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

Be sure you circle final answers!