Warm-Up to 6.5 – Double & Half Angle Formulas

Given: $\cos(2\theta) = \cos(\theta + \theta)$

Using what trig formulas you know so far, determine at least 2 solutions to this problem. Using your whiteboard, hold up your final solutions to

$\cos(2\theta) =$ ____________________

$\cos(2\theta) =$ ____________________

Solution:

$\cos(2\theta) = \cos(\theta + \theta)$	(1) Original problem
$\cos(2\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$	(2) Double angle formula substitution
$\cos(2\theta) = \cos^2\theta - \sin^2\theta$	(3) Simplify
$\cos(2\theta) = (1 - \sin^2\theta) - \sin^2\theta$	(4) Trig substitution for $\cos^2\theta$
$= 1 - 2\sin^2\theta$	(5) Simplify
$\cos(2\theta) = \cos^2\theta - \sin^2\theta$	Same as (3) above
$= \cos^2\theta - (1 - \cos^2\theta)$	(6) Trig substitution for $\sin^2\theta$
$= 2\cos^2\theta - 1$	(7) Simplify

Add these to your notes:

6.5 THE DOUBLE-ANGLE IDENTITIES:

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$

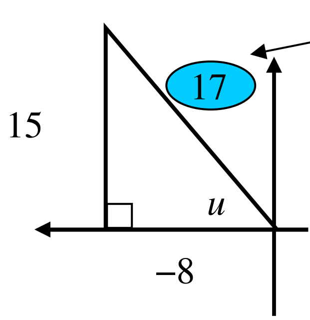
$$\tan(2\alpha) = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

Given: $\tan u = \frac{-15}{8}$, and $\frac{\pi}{2} < u < \pi$

Find the **EXACT** values of $\sin 2u$, $\cos 2u$, *and* $\tan 2u$ using the double-angle formulas.

Solution:

(1) Draw picture for the situation & find missing side by using *Pythagorean Theorem*



! Pay attention to the sign of your numbers!

$\sin(2u) = 2\sin u\cos u$ (2) Write the formula

$= 2\left(\frac{15}{17}\right)\left(\frac{-8}{17}\right)$ (3) Substitute in formula

$= \frac{-240}{289}$ (4) Simplify

$\text{Cos}(2u) = \cos^2 u - \sin^2 u$

$= \left(\frac{-8}{17}\right)^2 - \left(\frac{15}{17}\right)^2$

$= \frac{64}{289} - \frac{225}{289}$

$= \frac{-161}{289}$

$\tan(2u) = \frac{2\tan u}{1-\tan^2 u} = \frac{2\left(\frac{-15}{8}\right)}{1-\left(\frac{-15}{8}\right)^2} = \frac{\left(\frac{-30}{8}\right)}{1-\left(\frac{225}{64}\right)}$

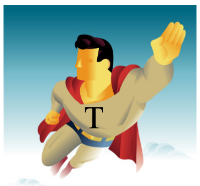
$= \frac{\left(\frac{-30}{8}\right)}{\frac{64}{64}-\left(\frac{225}{64}\right)} = \frac{\left(\frac{-30}{8}\right)}{\left(\frac{-161}{64}\right)} = \frac{-30}{8} \div \frac{-161}{64}$

$= \frac{-30}{8} \bullet \frac{64}{-161}$

$= \frac{-30}{\cancel{8}\,1} \bullet \frac{\cancel{64}\,8}{-161} = \frac{240}{161}$

OH MY GOSH! You had an AP Gov test the same day as your math test. The period before!! Mr. Muscarella is so mean. Despite Stephen & Sarah's pleading, he didn't give you an index card or let you use a formula sheet, and he wouldn't move the test either …

You remember the formulas for sin (2u) and cos (2u), but can't remember the tan (2u) formula.



But wait! You do remember the quotient identity tan (u) = $\frac{\sin u}{\cos u}$.

Could this be your saving grace? Does tan (2u) = $\frac{\sin(2u)}{\cos(2u)}$?

Try it to see what you'd come up with using the values above.

<u>Solution:</u>

$$\tan(2u) = \frac{\sin(2u)}{\cos(2u)} = \frac{-240}{289} \div \frac{-161}{289} = \frac{-240}{\cancel{289}} \bullet \frac{\cancel{289}}{-161} = \frac{240}{161}$$

Can you come up with another way to represent each angle so that part of it could be on the unit circle?

Solve for x.

Ex] $22.5° = \frac{1}{2}x$	Ex] $\frac{-\pi}{12} = \frac{1}{2}x$
$45° = x$	$\frac{-\pi}{6} = x$

This is the idea behind…

Trig Time Gifts from Muscarella's Workshop, 2007

THE HALF-ANGLE IDENTITIES:

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} = \pm\sqrt{\frac{1}{2}(1-\cos u)}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}} = \pm\sqrt{\frac{1}{2}(1+\cos u)}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

*** The <u>signs</u> of sin $\frac{u}{2}$ and cos $\frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.*

Put your heads together to try to figure this out…

Draw a picture for each situation. Be ready to justify your reasoning.

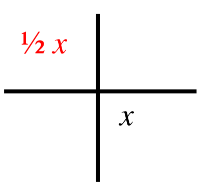
1) If an angle, x, is in the 4^{th} quadrant, then where is the angle (½ x) located?

2) If an angle, x, is in the 3^{rd} quadrant, then where is the angle (½ x) located?

3) If an angle, x, is in the 2^{nd} quadrant, then where is the angle (½ x) located?

<u>Solutions:</u>

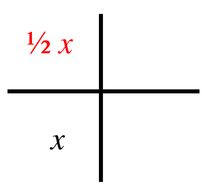
1)



$$\frac{3\pi}{2} < x < 2\pi$$

$$\Rightarrow \frac{3\pi}{4} < \frac{x}{2} < \pi$$

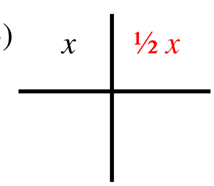
2)



$$\pi < x < \frac{3\pi}{2}$$

$$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

3)



$$\frac{\pi}{2} < x < \pi$$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Given: $\cos u = \frac{8}{17}$, $\frac{3\pi}{2} < u < 2\pi$

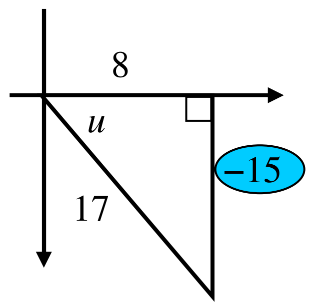
Find the **EXACT** values of $\sin\frac{u}{2}$, $\cos\frac{u}{2}$, and $\tan\frac{u}{2}$.

Solutions:

$\sin\frac{u}{2} = \frac{3\sqrt{34}}{34}$ $\cos\frac{u}{2} = \frac{-5\sqrt{34}}{34}$ $\tan\frac{u}{2} = \frac{-3}{5}$

Drawing:

! Careful! Since our triangle is in Quad IV, when we do the half angle formulas for sin and cos, be sure to get the right sign!!



You've Got Problems!

pp. 455 – 457: 7, 11, 17, 19, 21, 25, 29,
30 (use angle sum formulas, 3x = 2x +1x),
35, 41, 49