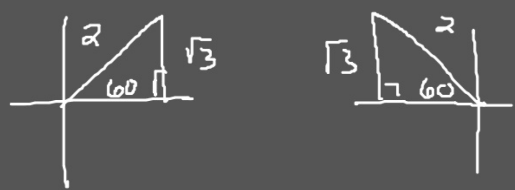
Solving Trig Equations
Review

1. What are all of the solutions to $5\sin x = 3\sin x + \sqrt{3}$?

$$2\sin x = \sqrt{3}$$

$$\sin x = \frac{\sqrt{3}}{2}$$



$\frac{\pi}{3}$ $\frac{2\pi}{3}$

$$\boxed{\begin{array}{l} \frac{\pi}{3} + 2\pi K, \ K \in \mathbb{Z} \\ \frac{2\pi}{3} + 2\pi K, \ K \in \mathbb{Z} \end{array}}$$

2. What are the solutions of $2\cos^2 x + \cos x - 1 = 0$ when $\theta \in [0, 2\pi)$?

let $u = \cos x$

$2u^2 + u - 1 = 0$

$(2u-1)(u+1) = 0$

$2u - 1 = 0$ $\quad$ $u + 1 = 0$

$u = \frac{1}{2}$ $\quad$ $u = -1$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad$ $x = \pi$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi}$

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$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ $\quad$ $\cos x - 1 = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = 1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad$ $x = \pi$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi}$

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3. Use a trig identity to solve $4\sin^2 x + 4\cos x - 5 = 0$ when $\theta \in [0, 2\pi)$

$4(1-\cos^2 x) + 4\cos x - 5 = 0$

$4 - 4\cos^2 x + 4\cos x - 5 = 0$

$-4\cos^2 x + 4\cos x - 1 = 0$

$4\cos^2 x - 4\cos x + 1 = 0$

let $u = \cos x$

$4u^2 - 4u + 1 = 0$

$(2u-1)(2u-1) = 0$

$2u - 1 = 0$

$u = 1/2$

$\cos x = \frac{1}{2}$

$\boxed{x = \pi/3,\ 5\pi/3}$

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$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

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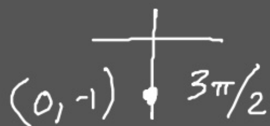
4. Solve the equation in the inveral [0, 2π).



$$\sin(\underline{3\theta}) = -1$$

$$3\theta = \frac{3\pi}{2} + 2\pi k, \quad k \in \mathbb{Z}$$

$$\theta = \frac{\pi}{2} + \frac{2\pi k}{3}, \quad k \in \mathbb{Z}$$

$$\theta = \frac{3\pi}{6} + \frac{4\pi k}{6}, \quad k \in \mathbb{Z}$$

$$k = 0 \qquad \frac{3\pi}{6} + \frac{4\pi(0)}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$$

$$k = 1 \qquad \frac{3\pi}{6} + \frac{4\pi(1)}{6} = \frac{7\pi}{6}$$

$$k = 2 \qquad \frac{3\pi}{6} + \frac{4\pi(2)}{6} = \frac{11\pi}{6}$$

$$2\pi = \frac{12\pi}{6}$$

$$k = 3 \qquad \frac{3\pi}{6} + \frac{4\pi(3)}{6} = \cancel{\frac{15\pi}{6}}$$

$$\boxed{\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

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5. Solve the equation in the inveral [0, 2π).

$$2\tan\theta\sin^2\theta = \tan\theta$$

$2\tan\theta \sin^2\theta - \tan\theta = 0$

$\tan\theta\,(2\sin^2\theta - 1) = 0$

$\tan\theta = 0$

$\theta = 0, \pi$

$2\sin^2\theta - 1 = 0$

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \pm\sqrt{\frac{1}{2}}$

$\sin\theta = \pm\frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\boxed{\theta = 0, \pi/4, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}}$

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6. Solve the equation in the inveral [0, 2π).

$$\cos 2x + 5\cos x + 3 = 0$$

$$(2\cos^2 x - 1) + 5\cos x + 3 = 0$$

$$2\cos^2 x + 5\cos x + 2 = 0$$

$$(2\cos x + 1)(\cos x + 2) = 0$$

$$2\cos x + 1 = 0 \qquad \cos x + 2 = 0 \qquad [-1, 1]$$

$$\cos x = \frac{-1}{2} \qquad \cancel{\cos x = -2}$$

$$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$$

6. Solve the equation in the inveral [0, 2π).

$$\cos 2x + 5\cos x + 3 = 0$$

$(2\cos^2 x - 1) + 5\cos x + 3 = 0$

$2\cos^2 x + 5\cos x + 2 = 0$

$(2\cos x + 1)(\cos x + 2) = 0$

$2\cos x + 1 = 0$ $\quad$ $\cos x + 2 = 0$

$\cos x = \frac{-1}{2}$ $\quad$ $\cos x = -2$

$$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$$

let $u = \cos x$

$2u^2 + 5u + 2 = 0$

$(2u+1)(u+2) = 0$

$2u + 1 = 0$ $\quad$ $u + 2 = 0$

$u = -1/2$ $\quad$ $u = -2$

$\cos x = -1/2$ $\quad$ $\cos x = -2$

$$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$$

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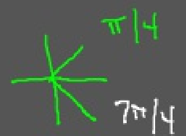
7. Solve the equation in the inveral [0, 2π).

$$\cos\Big(\underset{\alpha}{5x}\Big)\cos\Big(\underset{\beta}{x}\Big)+\sin\Big(\underset{\alpha}{5x}\Big)\sin\Big(\underset{\beta}{x}\Big)=\frac{\sqrt{2}}{2}$$

$$\cos(5x-x)=\frac{\sqrt{2}}{2}$$

$$\cos(4x)=\frac{\sqrt{2}}{2}$$



$4x=\pi/4+2\pi k,\ k\in\mathbb{Z}$ $\qquad$ $4x=\frac{7\pi}{4}+2\pi k,\ k\in\mathbb{Z}$ $\qquad$ $2\pi\to\frac{32\pi}{16}$

$x=\frac{\pi}{16}+\frac{\pi}{2}k,\ k\in\mathbb{Z}$ $\qquad$ $x=\frac{7\pi}{16}+\frac{\pi}{2}k,\ k\in\mathbb{Z}$

$x=\frac{\pi}{16}+\frac{8\pi}{16}k,\ k\in\mathbb{Z}$ $\qquad$ $x=\frac{7\pi}{16}+\frac{8\pi}{16}k,\ k\in\mathbb{Z}$

$k=0$ $\quad x=\pi/16$ $\qquad$ $x=\frac{7\pi}{16}$

$k=1$ $\quad x=\frac{\pi}{16}+\frac{8\pi}{16}(1)=\frac{9\pi}{16}$ $\qquad$ $x=\frac{7\pi}{16}+\frac{8\pi}{16}(1)=\frac{15\pi}{16}$

$k=2$ $\quad x=\frac{\pi}{16}+\frac{8\pi}{16}(2)=\frac{17\pi}{16}$ $\qquad$ $x=\frac{7\pi}{16}+\frac{8\pi}{16}(2)=\frac{23\pi}{16}$

$k=3$ $\quad x=\frac{\pi}{16}+\frac{8\pi}{16}(3)=\frac{25\pi}{16}$ $\qquad$ $x=\frac{7\pi}{16}+\frac{8\pi}{16}(3)=\frac{31\pi}{16}$

$k=4$ $\quad x=\frac{\pi}{16}+\frac{8\pi}{16}(4)=\frac{35\pi}{16}$ $\qquad$ $x=\frac{7\pi}{16}+\frac{8\pi}{16}(4)=\frac{39\pi}{16}$

$$\boxed{x=\frac{\pi}{16},\ \frac{7\pi}{16},\ \frac{9\pi}{16},\ \frac{15\pi}{16},\ \frac{17\pi}{16},\ \frac{23\pi}{16},\ \frac{25\pi}{16},\ \frac{31\pi}{16}}$$