Objective: Students will understand what it means to describe, graph and write the equation of a parabola.

## Parabolas

Parabola: collection of all points $P$ in a plane that are the same distance from a fixed point, the focus F, and a fixed line, the directrix $D$

$$
d(F, P)=d(P, D)
$$



Equation of a Parabola w/ vertex @ ( 0,0 ) \& focus @ $(a, 0), a>0$ is...

Vertex at (0, 0), a > 0

| Opens up | Opens down |
| :--- | :--- |
| Focus: | Focus: |
| Directrix: | Directrix: |
| Opens right | Opens left |
| Focus: | Focus: |
| Directrix: | Directrix: |

latus rectum - a segment that goes through the focus, and its endpoints are points on the parabola. The endpoints are a distance $\pm 2 a$ from the focus.

Ex 1 Discuss each equation. (So, find the vertex, focus and directrix.)

| a) $y^{2}=8 x$ | b) $x^{2}=-\frac{1}{2} y$ |
| :--- | :--- |

Ex 2 Find the equation of the parabola with vertex at $(0,0)$; axis of symmetry the $x$-axis; and contains the point $(2,3)$.

Vertex at (h,k)... Use patterns to describe the focus and directrix.

| Opens up | Opens down |
| :--- | :--- |
| Opens right | Opens left |
|  |  |

Ex 3 Find the equation of the parabola whose vertex is at $(4,-2)$ and focus is at $(6,-2)$.

Ex 4 Discuss the equation: $\quad y^{2}+12 y=-x+1$

Objective: Students will be able to graph, write the equation, identify key elements and convert between forms.

## Ellipses

Ellipse: the collection of all points in the plane the sum of whose distances from two fixed points, called foci, is a constant

- Major axis - the line containing the foci
- Center - the midpoint of the segment joining the foci
- Minor axis - the line that's through the center and perpendicular to the major axis
- Vertices - points at the intersection of the major axis and the ellipse
- Co-vertices - point at the intersection of the minor axis and the ellipse
- Standard Form - $\frac{(x-h)^{2}}{a^{2}}+\frac{(y-k)^{2}}{b^{2}}=1$
- General form $-a x^{2}+b y^{2}+c x+d y+e=0$.

Ex 1 Label the ellipse's major axis, center, minor axis, vertices and co-vertices.


Equation of an Ellipse: center @ $(0,0)$

Foci @ $( \pm c, 0)$ \& vertices @ $( \pm a, 0)$

$$
\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1
$$

where $a>b>0$ and $b^{2}=a^{2}-c^{2}$

Foci @ $(0, \pm c)$ \& vertices @ $(0, \pm a)$

$$
\frac{x^{2}}{b^{2}}+\frac{y^{2}}{a^{2}}=1
$$

where $a>b>0$ and $b^{2}=a^{2}-c^{2}$

FYI: "Discuss the equation" in ellipses means find the center, major axis, foci, vertices and co-vertices.

Ex 2 Graph the equation of the conic section. Find the vertices, co-vertices and foci.

$$
4 y^{2}+9 x^{2}=36
$$



Ex 3 Find an equation for each ellipse given...

| a) Center at $(0,0)$; focus at $(-1,0)$; vertex at $(-3,0)$ | b) Foci at $(0, \pm 2)$; major axis measures 8 |
| :--- | :--- |
|  |  |

Equation of an Ellipse: center @ $(h, k)$
Foci @ $(h \pm c, k)$ \& vertices @ $(h \pm a, k)$
Foci @ $(h, k \pm c)$ \& vertices @ $(h, k \pm a)$

$$
\frac{(x-h)^{2}}{a^{2}}+\frac{(y-k)^{2}}{b^{2}}=1
$$

where $a>b>0$ and $b^{2}=a^{2}-c^{2}$

$$
\frac{(x-h)^{2}}{b^{2}}+\frac{(y-k)^{2}}{a^{2}}=1
$$

where $a>b>0$ and $b^{2}=a^{2}-c^{2}$

Ex 4 Find an equation of an ellipse whose foci are $(1,2)$ and $(-3,2) \&$ whose vertex is $(-4,2)$.

Ex 5 Discuss each equation. (So, find the center, foci, vertices, co-vertices.)

| a) $9(x-3)^{2}+(y+2)^{2}=18$ | b) $4 x^{2}+3 y^{2}+8 x-6 y=5$ |
| :--- | :--- |
|  |  |

How would you find the $x$-intercept of any given ellipse? How would you find the $y$-intercept of any ellipse?

Objective: Students will be able to describe, write the equation of and graph hyperbolas.

## Hyperbolas

Hyperbola: the collection of all points in the plane the difference of whose distances from two fixed points, called foci, is a constant

- Transverse axis - the line containing the foci
- Center - the midpoint of the segment joining the foci
- Conjugate axis - the line through the center \& perpendicular to the transverse axis
- Branches - two separate curves which make the hyperbola's graph. They are symmetrical with respect to the transverse axis.
- Vertices - points at the intersection of the major axis and the hyperbola

Ex 1 Label the axes, center, branches, and vertices of the hyperbola.


Equation of a Hyperbola: center @ $(0,0)$

| Foci @ $( \pm c, 0)$ \& vertices @ $( \pm a, 0)$ | Foci @ $(0, \pm c)$ \& vertices @ $(0, \pm a)$ |
| :---: | :---: |
| Transverse axis parallel to x-axis | Transverse axis parallel to y-axis |
| $\frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1$ | $\frac{y^{2}}{a^{2}}-\frac{x^{2}}{b^{2}}=1$ |
| where $b^{2}=c^{2}-a^{2}$ | where $b^{2}=c^{2}-a^{2}$ |
| Note: Asymptotes are $y= \pm \frac{b}{a} x$. | Note: Asymptotes are $y= \pm \frac{a}{b} x$. |

FYI: "Discuss the equation" for hyperbolas means that you'll find the center, transverse axis, foci, and vertices that hyperbola.

Remember: How would you find the x-intercepts and y-intercepts of a hyperbola?

Ex 2 Graph the hyperbola. Then, find its center, foci, vertices and asymptotes.

$$
\frac{x^{2}}{4}-\frac{y^{2}}{25}=1
$$



Equation of a Hyperbola: center @ (h, k)
Foci @ $(h \pm c, k)$ \& vertices @ $(h \pm a, k)$
Transverse axis parallel to $x$-axis

$$
\begin{gathered}
\frac{(x-h)^{2}}{a^{2}}-\frac{(y-k)^{2}}{b^{2}}=1 \\
\text { where } b^{2}=c^{2}-a^{2}
\end{gathered}
$$

Note: Asymptotes are $(y-k)= \pm \frac{b}{a}(x-h)$.

Foci @ $(h, k \pm c)$ \& vertices @ $(h, k \pm a)$
Transverse axis parallel to $y$-axis

$$
\begin{gathered}
\frac{(y-k)^{2}}{a^{2}}-\frac{(x-h)^{2}}{b^{2}}=1 \\
\text { where } b^{2}=c^{2}-a^{2}
\end{gathered}
$$

Note: Asymptotes are $(y-k)= \pm \frac{a}{b}(x-h)$.

Ex 4 Find an equation for a hyperbola whose vertices are $(4,0)$ and $(-4,0)$ and has an asymptote of $y=2 x$. Then state its foci

Ex 5 Find an equation for a hyperbola whose center is at $(-3,1)$, focus is at $(-3,6)$ and whose vertex is at $(-3,4)$.

Ex 6 Find an equation for a hyperbola whose vertices are at $(1,-3)$ and $(1,1)$ and whose asymptote is $y+1=\frac{3}{2}(x-1)$.

