$x^2 + 9y^2 + 8x - 36y - 29 = 0$

$x^2 + 8x + 9y^2 - 36y = 29$

$(x^2 + 8x + 16) + 9(y^2 - 4y + 4) = 29 + 16 + 36$

$\frac{(x+4)^2}{81} + \frac{9(y-2)^2}{81} = \frac{81}{81}$

$$\frac{(x+4)^2}{81} + \frac{(y-2)^2}{9} = 1$$ Ellipse

center: $(-4, 2)$

vertices: $(-13, 2)(5, 2)$

co-vertices: $(-4, -1)(-4, 5)$

Foci: $(-4-6\sqrt{2}, 2)(-4+6\sqrt{2}, 2)$

$a^2 = 81 \quad a = 9$

$b^2 = 9 \quad b = 3$

$c^2 = a^2 - b^2$

$c^2 = 81 - 9$

$c^2 = 72$

$c = \pm\sqrt{72}$

$c = \pm 6\sqrt{2}$

$\frac{81}{72}$

64

$(-4 \pm \sqrt{72}, 2)$

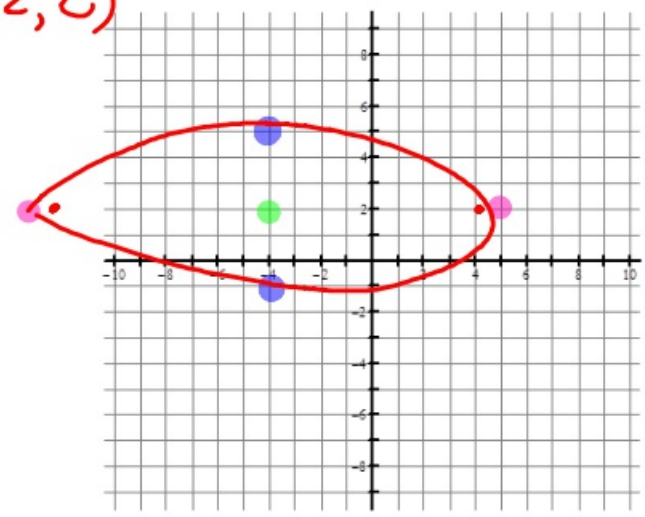
$\sqrt{9}\sqrt{8}$

$3\sqrt{8}$

$3\sqrt{4}\sqrt{2}$

$3 \cdot 2\sqrt{2}$

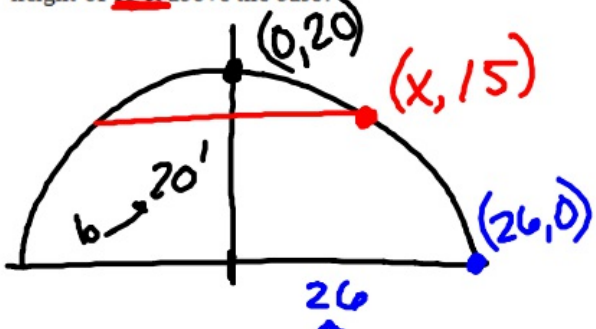
$6\sqrt{2}$



Application Problem

Include a drawing. Show all work that leads to your solution. Answer each question in a complete sentence. Be sure to include units in your final answers!

5. An arch is in the form of a semi-ellipse is 52 ft at the base and has a height of 20 ft. How wide is the arch at a height of 15 ft above the base?



a

52

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{26^2} + \frac{15^2}{20^2} = 1$

$\frac{x^2}{676} = 1 - \frac{225}{400}$

$\frac{x^2}{676} = \frac{400}{400} - \frac{225}{400}$

$\frac{x^2}{676} = \frac{175}{400}$

$x^2 = \left(\frac{175}{400}\right)676$

$x = \pm\sqrt{\left(\frac{175}{400}\right)(676)}$

$x \approx \pm 17.197$

x2

The arch is about 34ft wide 15 ft above the base.

$$4x^2 - y^2 + 8x - 2y - 13 = 0$$

$$4(x^2+2x+\underline{1}) - 1(y^2+2y+\underline{1}) = 13+\underline{4}+\underline{-1}$$

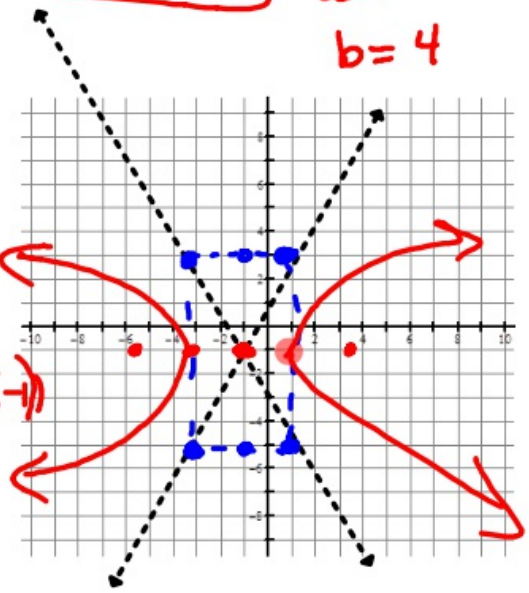
$$4(x+1)^2 - (y+1)^2 = 16$$

$$\frac{(x+1)^2}{4} - \frac{(y+1)^2}{16} = 1$$

H ><

$a = 2$

$b = 4$



center: $(-1,-1)$

foci $c^2 = a^2+b^2$

$c^2 = 20$

$c = \pm 2\sqrt{5}$

$(-1-2\sqrt{5},-1)$ $(-1+2\sqrt{5},-1)$

vertices: $(-3,-1)$ $(1,-1)$

$y - y_1 = m(x-x_1)$

$y+1 = 2(x+1)$ $y+1 = -2(x+1)$

$3y^2 + x - 6y = 2$

$3(y^2 - 2y + \underline{1}) = -x + 2 \underline{+3}$

$3(y-1)^2 = -x + 5$

$3(y-1)^2 = -1(x-5)$

$(y-1)^2 = -\frac{1}{3}(x-5)$

$V: (5, 1)$

$F\left(\frac{23}{12}, 1\right)$

$D \quad 5 + \frac{1}{12}$

$x = 61/12$

$-4a = -\frac{1}{3}$

$a = \frac{1}{12}$

$5 - \frac{1}{12}$

$\frac{60}{12} - \frac{1}{12}$

Problems from Quiz 1

$$x^2 + y^2 - 2x + 4y + 2 = 0$$

$$x^2 - 2x + \underline{1} + y^2 + 4y + \underline{4} = -2 + \underline{1} + \underline{4}$$

$$\boxed{(x-1)^2 + (y+2)^2 = 3}$$

Circle Center: $(1, -2)$

Radius: $\sqrt{3}$

$$49x^2 + 9y^2 + 294x = 0$$

$$49x^2 + 294x + 9y^2 = 0$$

$$49(x^2 + 6x + 9) + 9y^2 = 441$$

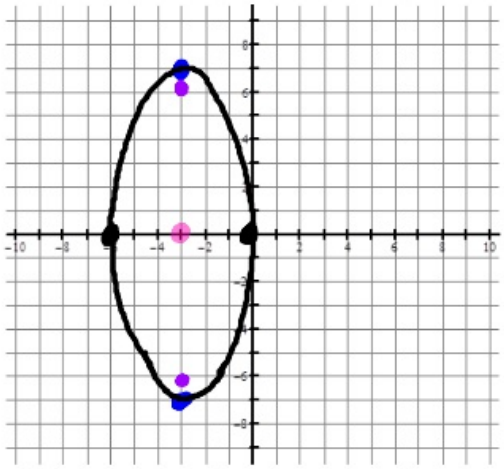
$$49(x+3)^2 + 9y^2 = 441$$

$$\frac{(x+3)^2}{9} + \frac{y^2}{49} = 1$$

$b = 3 \qquad a = 7$

$$c^2 = a^2 - b^2$$
$$c^2 = 49 - 9$$
$$c^2 = 40$$
$$c = \pm\sqrt{40} = \pm\sqrt{4}\sqrt{10}$$
$$c = \pm 2\sqrt{10}$$

→ more than 6, less than 7



Center: $(-3, 0)$

Vertices: $(-3, 7)\ (-3, -7)$

Co-vertices: $(0, 0)\ (-6, 0)$

foci: $(-3, 2\sqrt{10})$ $(-3, -2\sqrt{10})$

$$3y^2 + x - 6y - 2 = 0$$

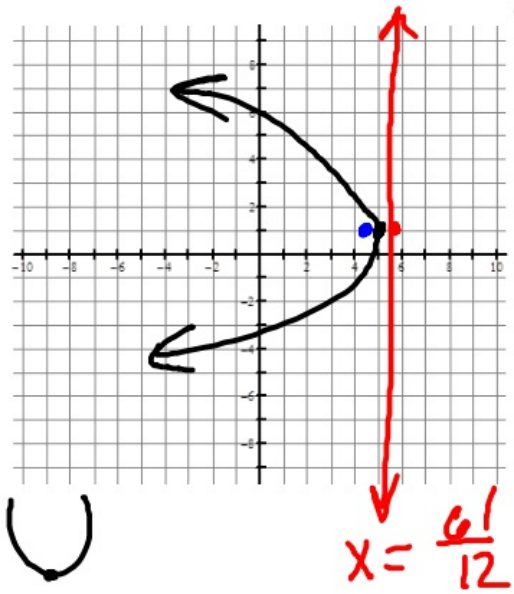
$$3y^2 - 6y = -x + 2$$

$$3(y^2 - 2y + \underline{1}) = -x + 2 + \underline{3}$$

$$3(y-1)^2 = -x + 5$$

$$3(y-1)^2 = -(x-5)$$

$$\boxed{(y-1)^2 = -\tfrac{1}{3}(x-5)}$$



$$-4a = \frac{-1}{3}$$

$$a = \frac{1}{12}$$

$V: (5, 1)$

$F: \left(5 - \frac{1}{12}, 1\right)$

$\left(\frac{60}{12} - \frac{1}{12}, 1\right)$

$\left(\frac{59}{12}, 1\right)$

D: $5 + \frac{1}{12}$

$\frac{60}{12} + \frac{1}{12}$

$\boxed{x = \frac{61}{12}}$

$$-x^2 + 2y^2 + 4y - 8 = 0$$

$$-x^2 + 2(y^2 + 2y + \underline{1}) = 8 + \underline{2}$$

$$-x^2 + 2(y+1)^2 = 10$$

$$\frac{(y+1)^2}{5} - \frac{x^2}{10} = 1$$

U

∩

$a = \sqrt{5}$

$b = \sqrt{10}$

$c^2 = a^2 + b^2$

$c^2 = 5 + 10$

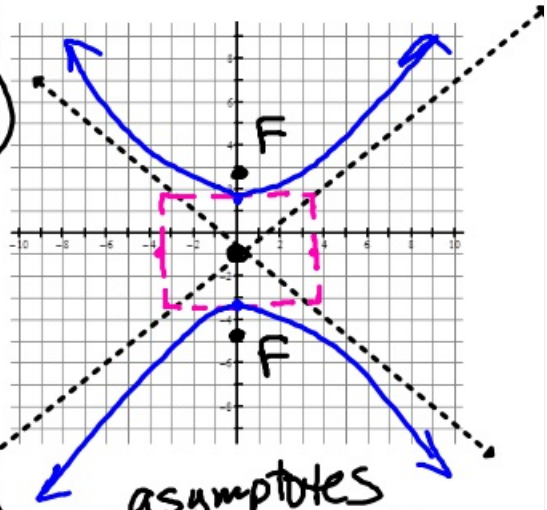
$c = \pm\sqrt{15}$

center: $(0, -1)$

vertices

$(0, -1+\sqrt{5})$

$(0, -1-\sqrt{5})$



Foci

$(0, -1+\sqrt{15})$

$(0, -1-\sqrt{15})$

asymptotes

$(y-K) = \pm\frac{a}{b}(x-h)$

$y+1 = \pm\frac{\sqrt{5}}{\sqrt{10}}(x-0)$

$y+1 = \frac{\sqrt{2}}{2}x$

$y+1 = -\frac{\sqrt{2}}{2}x$

$\frac{\sqrt{5}}{\sqrt{10}} = \sqrt{\frac{5}{10}} = \sqrt{\frac{1}{2}}$

$\frac{\sqrt{1}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$