

$$49x^2 + y^2 - 49 = 0$$

$$49x^2 + y^2 = 49$$

Ellipse

$$20x^2 + 20y^2 = 100$$

Circle

$$x^2 + y^2 = 5$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{> C}$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad \text{> C}$$

$$-4x^2 + 9y^2 + 8x - 18y - 31 = 0$$

#8, Conics
Rev. WS.

$$-4x^2 + 8x + 9y^2 - 18y = 31$$

$$-4(x^2 - 2x + \frac{1}{4}) + 9(y^2 - 2y + 1) = 31 + \frac{-4}{4} + \frac{9}{9}$$

$\left[\frac{1}{2}(-2)\right]^2 = (-1)^2 = 1$ $\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$

$$-4(x-1)^2 + 9(y-1)^2 = 36$$

$$\frac{9(y-1)^2}{36} - \frac{4(x-1)^2}{36} = \frac{36}{36}$$

Hyperbola



$$\frac{(y-1)^2}{4} - \frac{(x-1)^2}{9} = 1$$

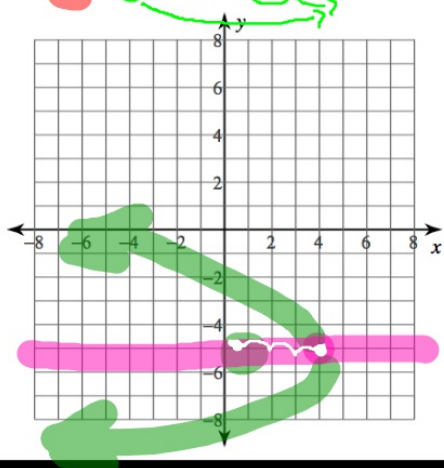
$$a^2 = 4$$

$$a = 2$$

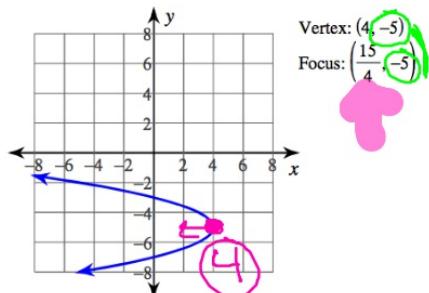
$$b^2 = 9$$

$$b = 3$$

16) $y^2 + x + 10y + 21 = 0$



16)



$$y^2 + 10y = -x - 21 \quad \left(\frac{\quad}{\quad}\right)^2 = \dots$$

$$\left(\frac{1}{2}10\right)^2 = (5)^2 = 25$$

$$y^2 + 10y + 25 = -x - 21 + 25$$

$$(y+5)^2 = -x + 4 \quad (h, k)$$

$$(y+5)^2 - 4 = -x$$

$$-(y+5)^2 + 4 = x$$

$$\Rightarrow \quad k \quad h$$

$$(y-k)^2 = -4a(x-h)$$

$$\Rightarrow (y+5)^2 = -x + 4$$

$$(y+5)^2 = -1(x-4) \quad -(x-h)$$

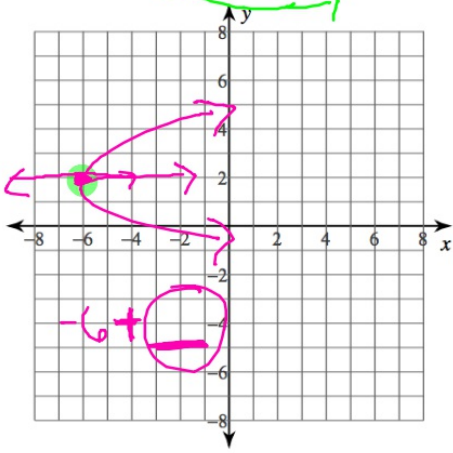
$$4a = -1$$

$$a = -1/4$$

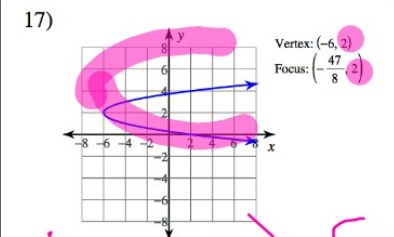
x-coord: $4 - \frac{1}{4}$
 $\frac{16}{4} - \frac{1}{4}$

$\frac{15}{4} \Rightarrow$ gives x-coord. of focus \Rightarrow y-coord is SAME AS vertex

17) $-2y^2 + x + 8y - 2 = 0$



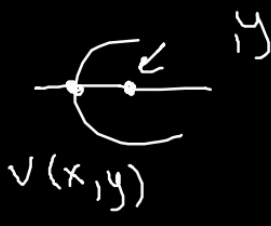
$$\begin{aligned}
 -2y^2 + 8y &= -x + 2 \\
 -2(y^2 - 4y) &= -x + 2 \\
 -2(y^2 - 4y + 4) &= -x + 2 - 8 \\
 -2(y - 2)^2 &= -x - 6 \\
 \rightarrow 2(y - 2)^2 - 6 &= x \quad (b, k) = (-6, 2)
 \end{aligned}$$



$(-\frac{47}{8}, 2)$ Focus

$$\begin{aligned}
 2(y - 2)^2 &= x + 6 \\
 (y - 2)^2 &= \left(\frac{1}{2}x\right) + 3 \\
 (y - 2)^2 &= \frac{1}{2}(x + 6) \quad \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12} \\
 -4a &= \frac{1}{2} \\
 a &= -\frac{1}{8}
 \end{aligned}$$

$$\begin{aligned}
 \rightarrow \\
 -6 + \frac{1}{8} \\
 -\frac{47}{8} + \frac{1}{8} \\
 -\frac{46}{8}
 \end{aligned}$$



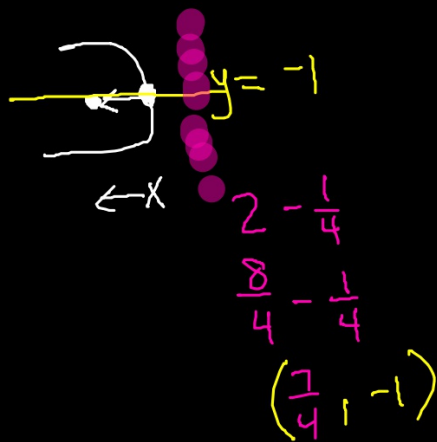
$$1) y^2 + x + 2y - 1 = 0$$

1) Parabola

$$x = -(y + 1)^2 + 2$$

Vertex: (2, -1)

Focus: $(\frac{7}{4}, -1)$



$$y^2 + 2y = -x + 1$$

$$y^2 + 2y + \frac{1}{4} = -x + 1 + \frac{1}{4}$$

$$\left(\frac{2}{2}\right)^2 = (1)^2 = 1$$

$$(y + 1)^2 = -x + 2$$

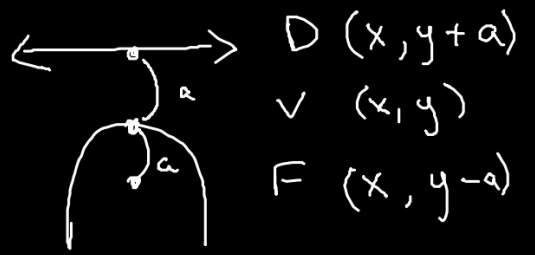
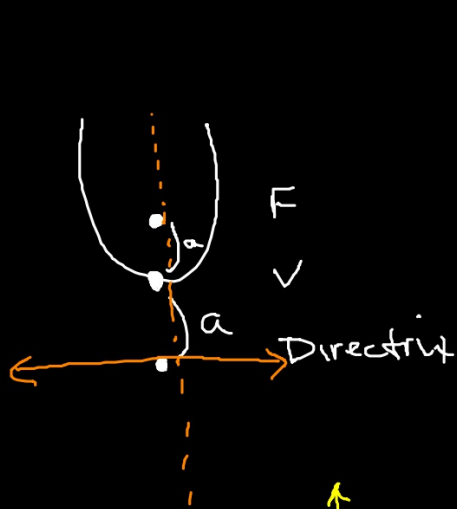
$$(y + 1)^2 = -1(x - 2)$$

$$-1 = 4a$$

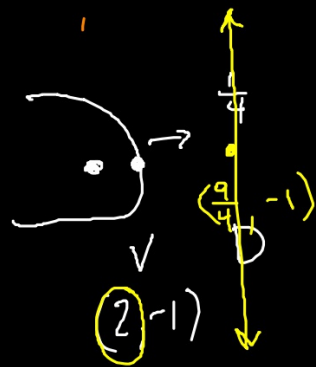
$$-\frac{1}{4} = a$$

$$y = 2x + 3$$

$$\frac{y - 3}{2} = x$$



$D (x, y+a)$
 $V (x, y)$
 $F (x, y-a)$



$$2 + \frac{1}{4}$$

$$\frac{8}{4} + \frac{1}{4}$$

$$x = \frac{9}{4} \quad \text{Directrix}$$

$$10) y^2 + x - 6y + 6 = 0$$

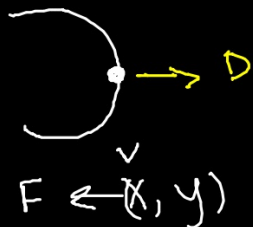
10) Parabola

$$x = -(y - 3)^2 + 3$$

Vertex: (3, 3)

Focus: $\left(\frac{11}{4}, 3\right)$

Latus Rectum: 1 unit



$$3 - \frac{1}{4}$$

$$\frac{12}{9} - \frac{1}{4}$$

$$F \left(\frac{11}{4}, 3 \right)$$

$$y^2 - 6y = -x - 6$$

$$y^2 - 6y + 9 = -x - 6 + 9$$

$$(y - 3)^2 = -x + 3$$

$$(y - 3) = \underbrace{-1}_{4a} (x - 3)$$

$$-1 = 4a$$

$$-\frac{1}{4} = a$$

Directrix

$$3 + \frac{1}{4}$$

$$\frac{12}{4} + \frac{1}{4}$$

$$x = \frac{13}{4}$$

$$8) -4x^2 + 9y^2 + 8x - 18y - 31 = 0$$

Hyperbola

$$\frac{(y-1)^2}{4} - \frac{(x-1)^2}{9} = 1$$

Center: (1, 1)

Vertices: (1, 3), (1, -1)

Foci: (1, 1 + $\sqrt{13}$), (1, 1 - $\sqrt{13}$)

Latus Rectum: 9 units

$$-4x^2 + 8x + 9y^2 - 18y = 31$$

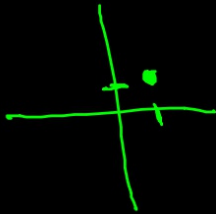
$$-4(x^2 - 2x + 1) + 9(y^2 - 2y + 1) = 31 - 4 + 9$$

$$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1 \quad \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$$

$$-4(x-1)^2 + 9(y-1)^2 = 36$$

$$\frac{9(y-1)^2}{36} - \frac{4(x-1)^2}{36} = \frac{36}{36}$$

$$\boxed{\frac{(y-1)^2}{4} - \frac{(x-1)^2}{9} = 1}$$



$$\frac{(y-1)^2}{4} - \frac{(x-1)^2}{9} = 1$$

$$a^2 = 4 \quad b^2 = 9 \quad c^2 = 16$$

$$a = 2 \quad b = 3 \quad c = 4$$

$$b^2 = c^2 - a^2$$

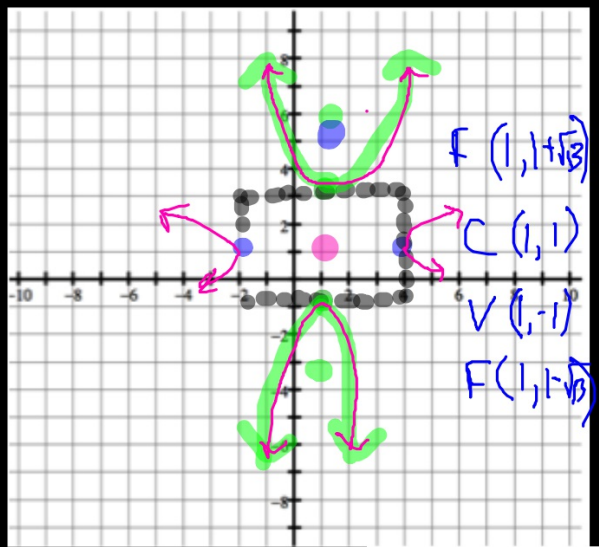
$$a^2 + b^2 = c^2$$

$$4 + 9 = c^2$$

$$\pm \sqrt{13} = c$$

$$c^2 = 16 \quad c^2 = 9$$

$$c = 4 \quad c = 3$$



Hyperbola

$$\frac{(y-1)^2}{4} - \frac{(x-1)^2}{9} = 1$$

 Center: $(1, 1)$
 Vertices: $(1, 3), (1, -1)$
 Foci: $(1, 1 + \sqrt{13}), (1, 1 - \sqrt{13})$
 Latus Rectum: 9 units

$$15) x^2 - y^2 + 2x - 2y - 15 = 0$$

15) Hyperbola

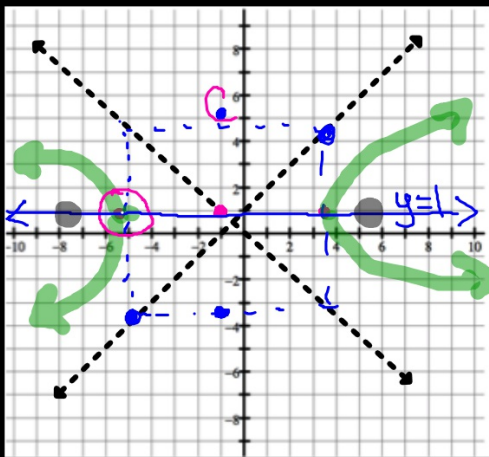
$$\frac{(x+1)^2}{17} - \frac{(y+1)^2}{17} = 1$$

Center: $(-1, -1)$

Vertices: $(-1 + \sqrt{17}, -1), (-1 - \sqrt{17}, -1)$

Foci: $(-1 + \sqrt{30}, -1), (-1 - \sqrt{30}, -1)$

$C: (-1, 1)$



$$x^2 + 2x + 1 - y^2 - 2y + 1 = 15 + 1 + 1$$

$$(x+1)^2 - (y-1)^2 = 17$$

$$\frac{(x+1)^2}{17} - \frac{(y-1)^2}{17} = 1 \quad \supset C$$

$$a = b = \sqrt{17}$$

$$c^2 = a^2 + b^2$$

$$c^2 = 17 + 17$$

$$c = \sqrt{34}$$

$$\text{Foci } (-1 - \sqrt{34}, -1) \quad (-1 + \sqrt{34}, -1)$$