Objective: Students will learn the basics of conversion and radian measures

Angle: a shape or *measure* made between two rays that share a fixed endpoint/**vertex**

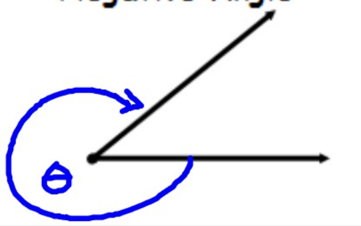
- **Initial Side** – the **fixed/stationary** ray (This will usually be on the positive x-axis.)
- **Terminal Side** – the ray that **rotates**
- **Positive Angle** – occurs when the terminal side rotates **counterclockwise**
- **Negative Angle** – occurs when the terminal side rotates **clockwise**
- **Standard Position** – the vertex of the angle is at the **origin** and the initial side is on the **positive x-axis**
- **Quadrantal Angle** – an angle whose initial side is in standard position and whose terminal side is on an **axis**
- **Co-terminal Angle:** an angle whose terminal side **coincides** with another angle's terminal side from standard position

Angle Measures: Two types – **degrees** and **radians**

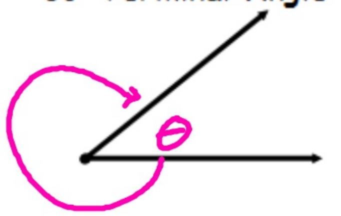
Drawing Angles

Positive Angle

Negative Angle



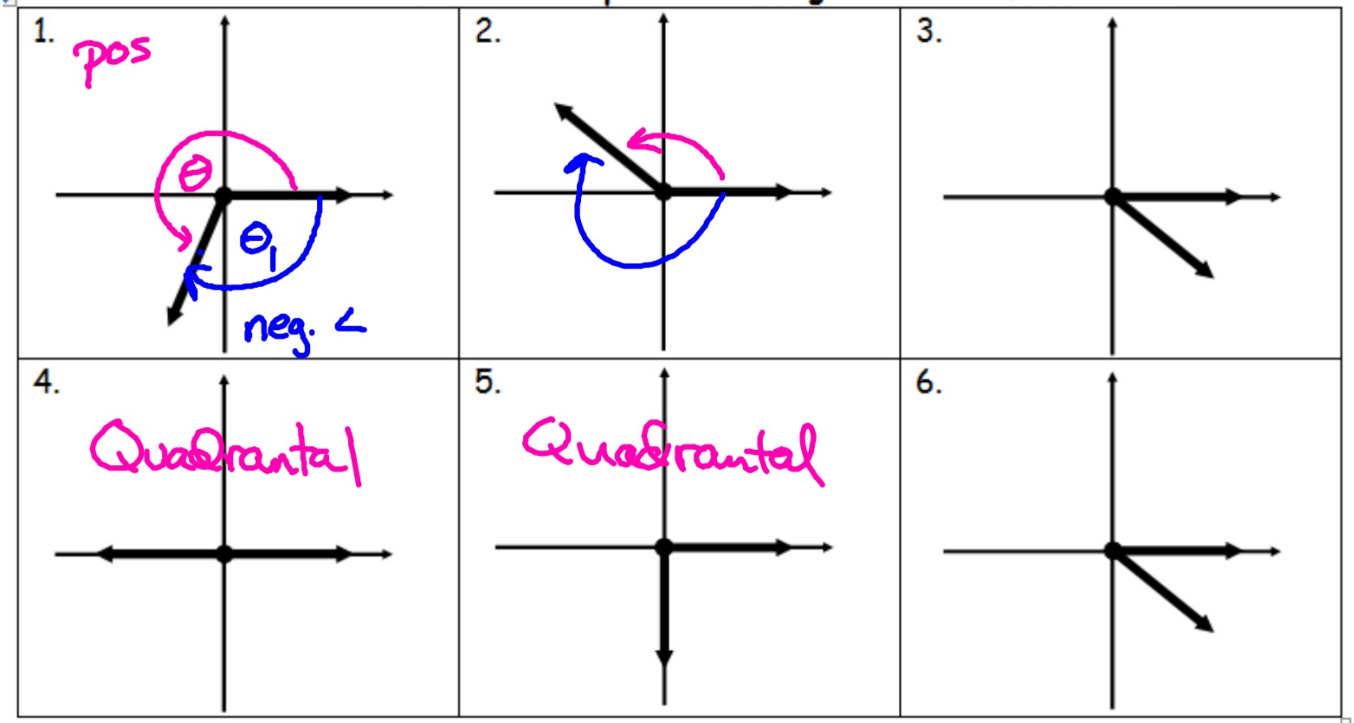
Co-Terminal Angle



45, -315

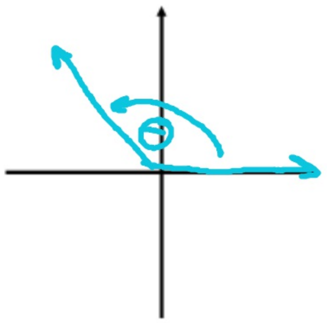
-45, 315

__Try__: Identify the angles as *positive*, *negative* or *quadrantal*. Identify the quadrant of the terminal side of θ, if possible. If the terminal side of θ is on an axis, state the axis and whether it is the positive or negative side of the axis.

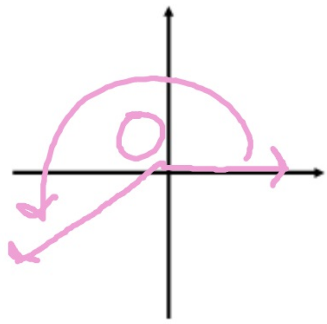


Try: Draw an angle from standard position that terminates in the given quadrant/axis

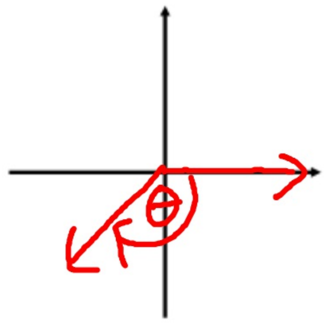
7. θ is positive and terminates in Quadrant 2



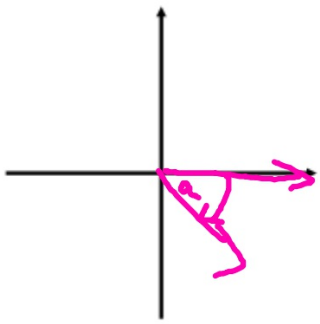
8. θ is positive and terminates in Quadrant 3



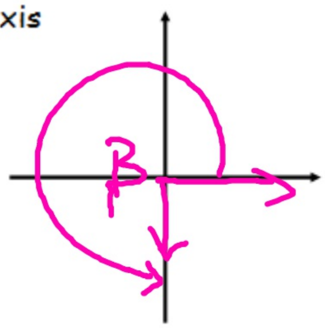
9. θ is negative and terminates in Quadrant 3



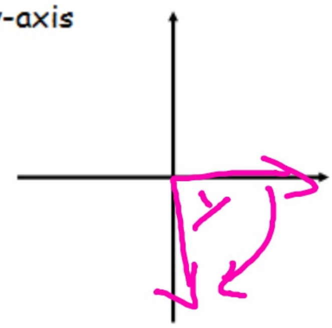
10. α is negative and terminates in Quadrant 4



11. β is positive and terminates on the negative y-axis



12. γ is negative and terminates on the negative y-axis



<u>Radians</u>: (Think of angles inside circles whose vertex is at the center of the circle.)

Central angle is an angle whose vertex is at the center of a circle.

One radian is the measure of the angle you get from a central angle when the arc on the circle is the same length as the radius of the circle.

The **radian** measure of an angle is the ratio of the arc length to the radius of a circle that the central angle intersects.

Theorem: For a circle of radius r, a central angle of θ radians subtends an arc whose length s is $r\theta$. $s = r\theta$

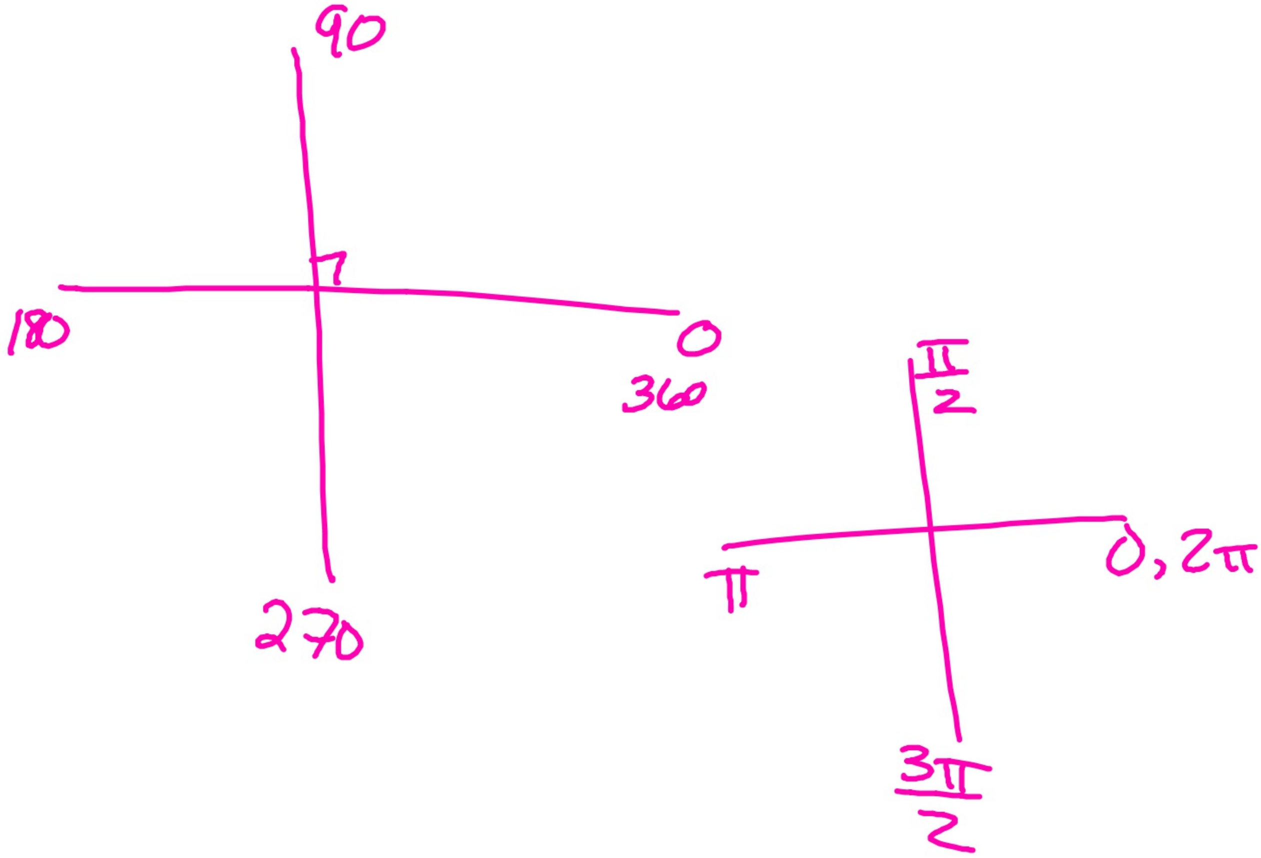
<u>Converting between Degrees & Radians</u>: 360° ⇨ Circumference ⇨ Arc Length

In essence…

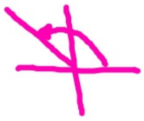
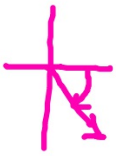
$360° = 2\pi$ radians; $180° = 1\pi$ radians; $1° = \frac{\pi}{180}$ radians; 1 radian = $\frac{180}{\pi}°$

Degrees to
RADIANS

$$\text{Angle} \times \frac{\pi}{180}$$


90
180
0
360
270
$\frac{\pi}{2}$
π
$0, 2\pi$
$\frac{3\pi}{2}$

Try: Convert each angle in degrees to radians. Show your conversion factor!

1. 120° $12\cancel{0} \times \frac{\pi}{18\cancel{0}} = \frac{2\pi}{3}$ 	2. 330° $33\cancel{0} \times \frac{\pi}{18\cancel{0}} = \frac{11\pi}{6}$
3. -30° $-30 \times \frac{\pi}{180} = -\frac{\pi}{6}$ 	4. -315° $-315 \times \frac{\pi}{180} = \frac{-7\pi}{4}$
5. -65° $-65 \times \frac{\pi}{180} = \frac{-13\pi}{36}$	6. 415° $415 \times \frac{\pi}{180} = \frac{83\pi}{36}$

$\boxed{\text{RADIANS to DEGREES}}$

$$\text{RAD} \times \frac{180}{\pi}$$

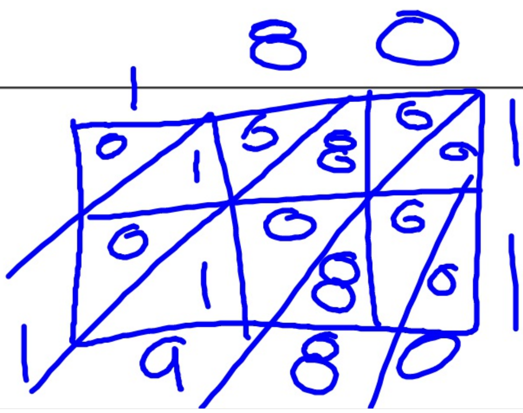
Try: Convert each angle in radians to degrees. Show your conversion factor!

7. $\frac{5\cancel{\pi}}{6} \cdot \frac{180}{\cancel{\pi}} = 150^\circ$	8. $\frac{5\cancel{\pi}}{4} \cdot \frac{180}{\cancel{\pi}} = \frac{5(90)}{2} = 225^\circ$
9. $\frac{5\pi}{3} \times \frac{180}{\pi} = 300^\circ$	10. $\frac{5\pi}{2} \cdot \frac{180}{\pi} = 450^\circ$
11. $\frac{7\pi}{\cancel{5}_{1}} \cdot \frac{\cancel{180}^{36}}{\pi} = 252^\circ$	12. $\frac{11\cancel{\pi}}{12} \cdot \frac{180}{\cancel{\pi}} = \frac{1980}{12} = 165^\circ$

11x15

165

$\frac{11\cancel{\pi}}{\cancel{12}_{\cancel{2}_{1}}} \cdot \frac{\cancel{180}^{\cancel{30}^{15}}}{\cancel{\pi}}$

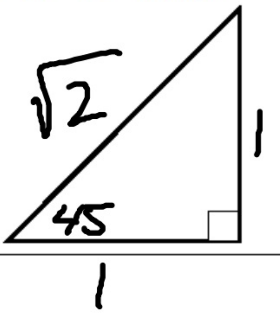


11×258

2838

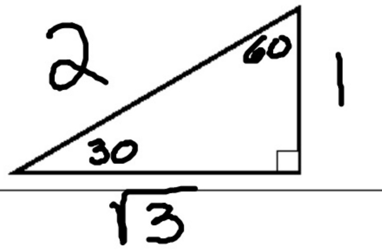
Corresponding Reference Triangles:

45°-45°-90° Triangle

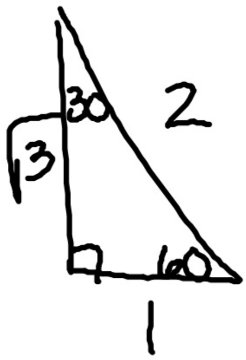


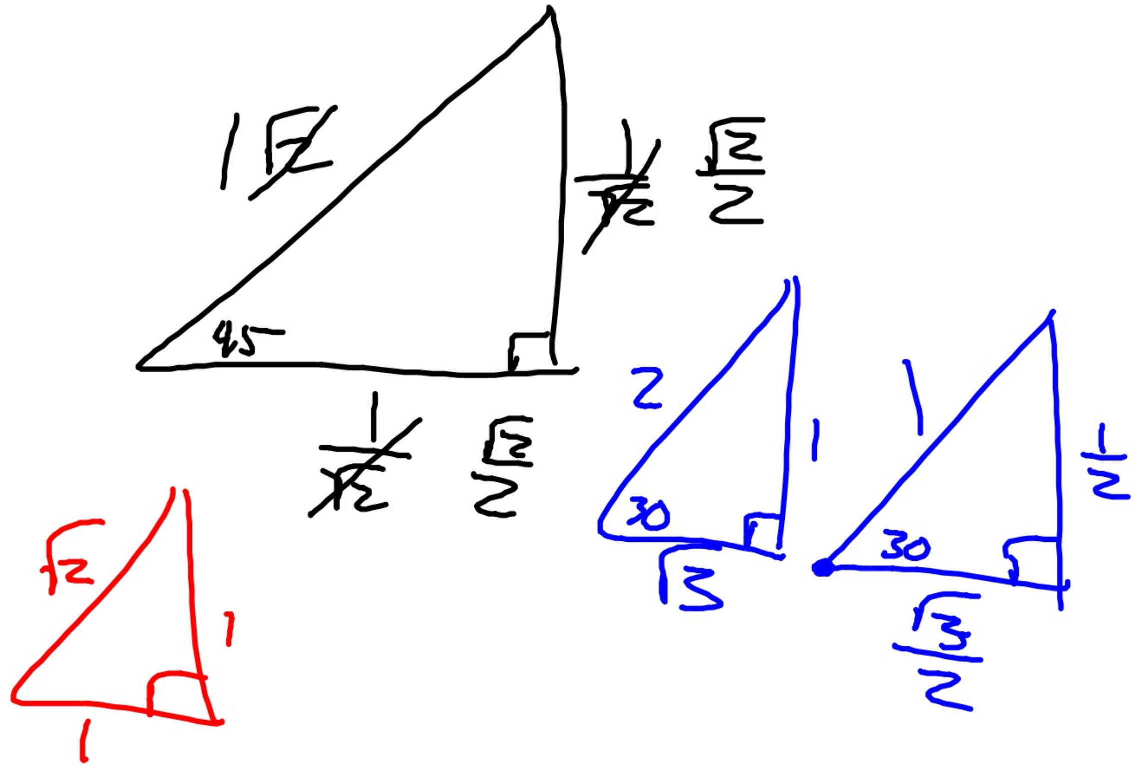
30°-60°-90° Triangle

☆ Note: Changing the orientation of this triangle makes either a 30° or a 60° reference angle.

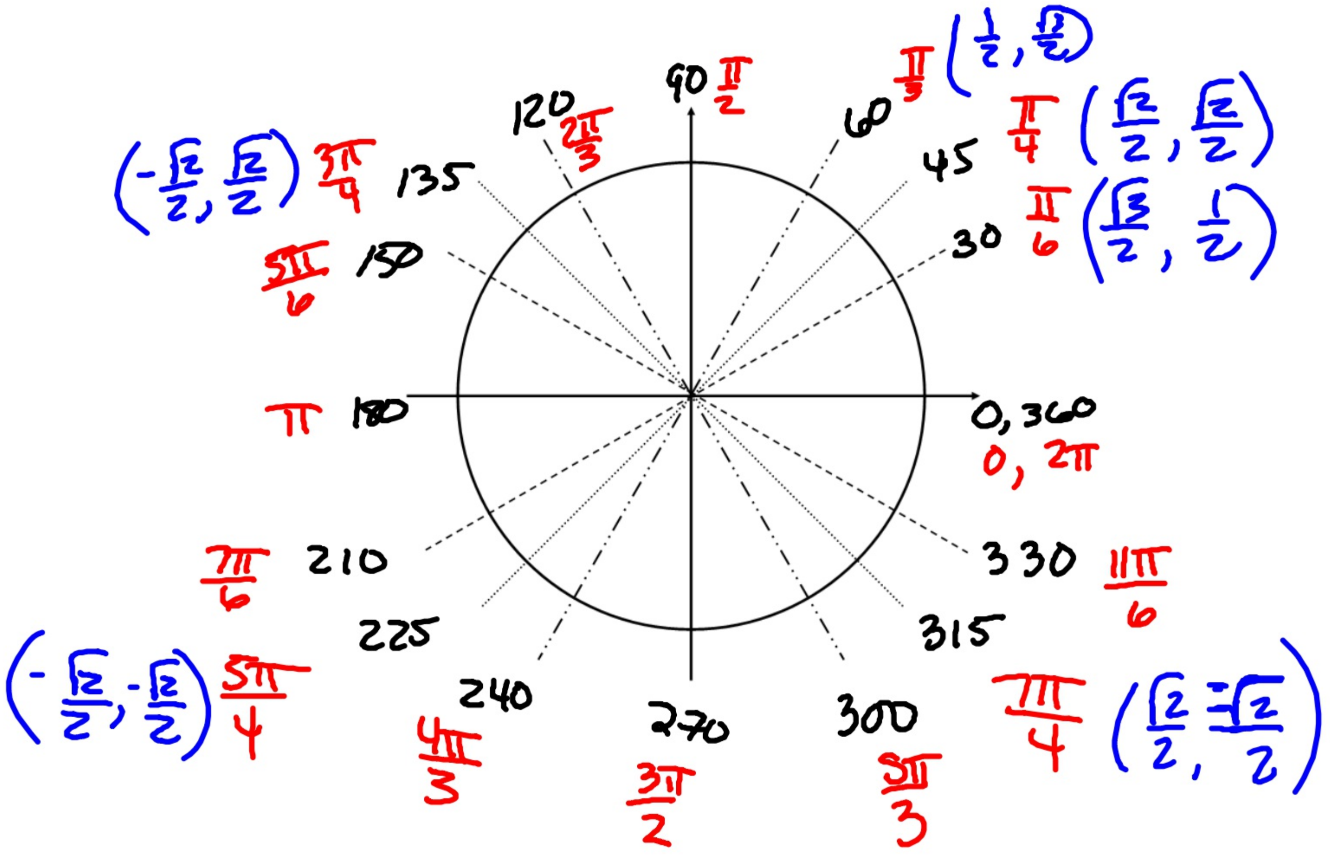


30, 60, 45

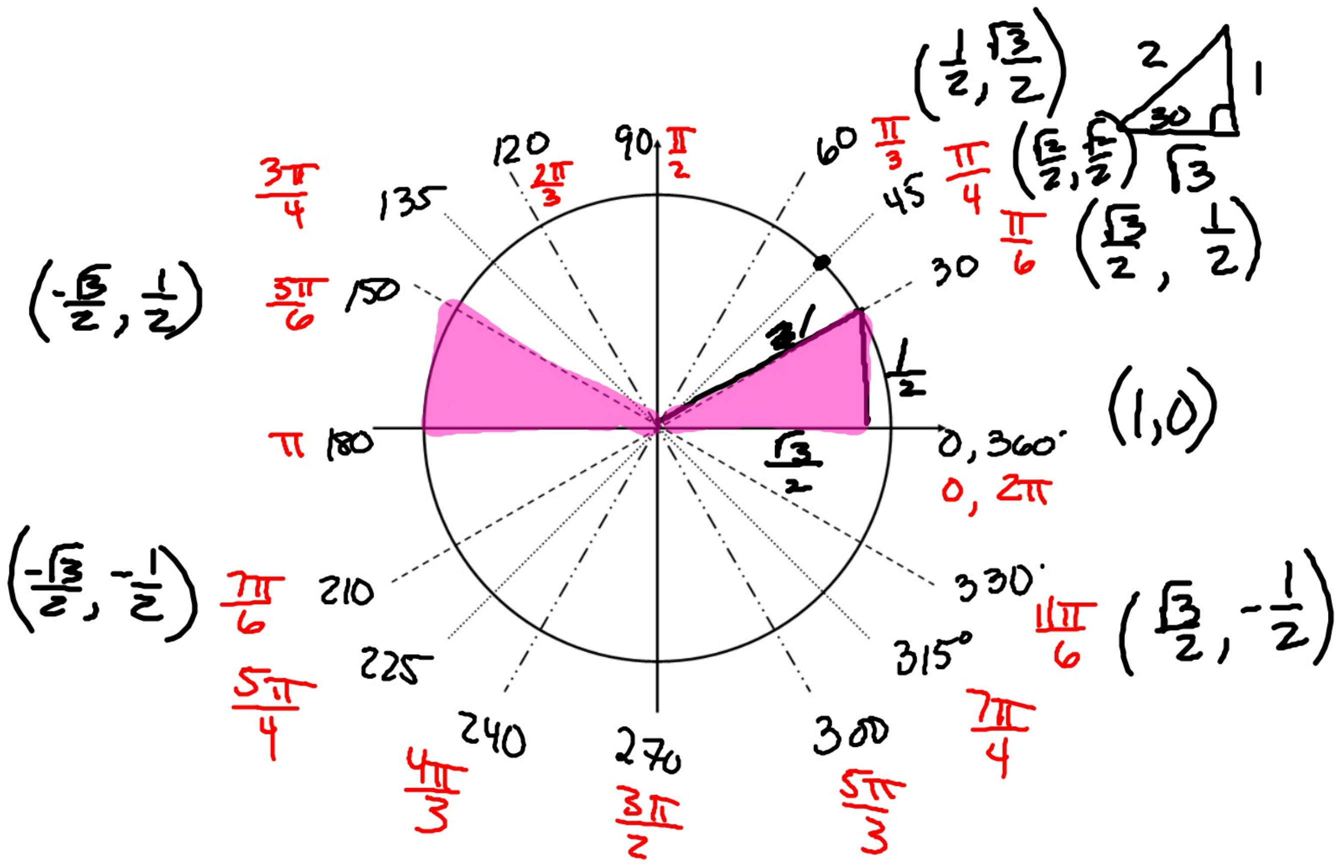




45
1
30
2
√2
√3
$\frac{1}{2}$
$\frac{\sqrt{2}}{2}$
$\frac{\sqrt{3}}{2}$

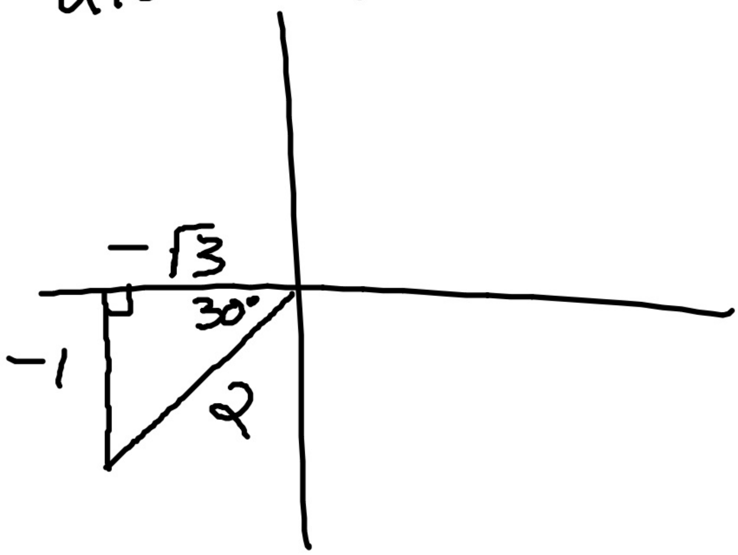


90 $\frac{\pi}{2}$
60 $\frac{\pi}{3}$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
120 $\frac{2\pi}{3}$
45 $\frac{\pi}{4}$ $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
135 $\frac{3\pi}{4}$ $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
30 $\frac{\pi}{6}$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
150 $\frac{5\pi}{6}$
180 π
0, 360
0, 2π
210 $\frac{7\pi}{6}$
330 $\frac{11\pi}{6}$
225 $\frac{5\pi}{4}$ $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
315 $\frac{7\pi}{4}$ $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
240 $\frac{4\pi}{3}$
270 $\frac{3\pi}{2}$
300 $\frac{5\pi}{3}$



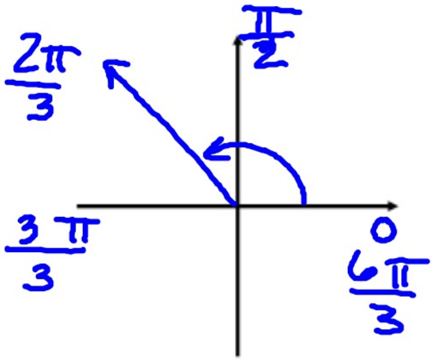
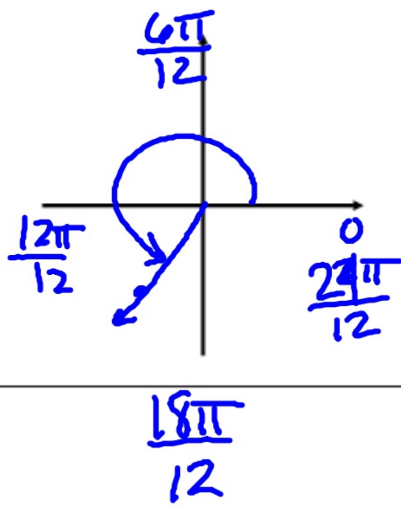
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
2
1
30
$\sqrt{3}$
90 $\frac{\pi}{2}$
120 $\frac{2\pi}{3}$
60 $\frac{\pi}{3}$
$\frac{3\pi}{4}$
135
45 $\frac{\pi}{4}$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
30 $\frac{\pi}{6}$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{5\pi}{6}$ 150
1
$\frac{1}{2}$
π 180
0, 360°
$\frac{\sqrt{3}}{2}$
0, 2π
(1,0)
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\frac{7\pi}{6}$ 210
330°
$\frac{11\pi}{6}$
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
225
$\frac{5\pi}{4}$
315°
$\frac{7\pi}{4}$
240 $\frac{4\pi}{3}$
270 $\frac{3\pi}{2}$
300 $\frac{5\pi}{3}$

$210° \rightarrow 30°$ Ref ∠.

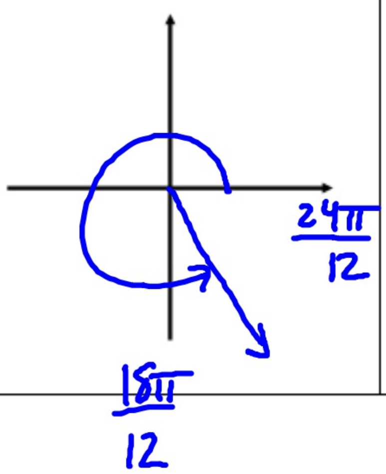
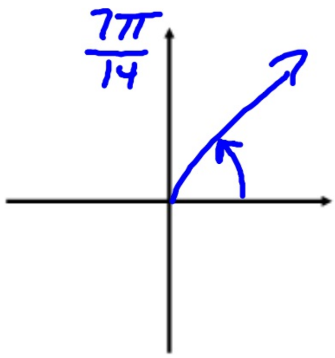


Example: Draw the angle in standard position with the given measure.

1. Start from standard position. Rotate in the positive or negative direction, depending on the sign of the angle.
2. Label each axis as fractions of π using the denominator to help you to locate the quadrant of the terminal side.
3. Visually subdivide the quadrant of the terminal side. Decide which axis will be closest to the terminal side.
4. Draw the terminal side and label the angle with its rotation.

A) $\frac{2\pi}{3}$ 	B) $\frac{17\pi}{12}$ 

Try: Draw the angle in standard position with the given measure.

13. $\frac{19\pi}{12}$ 	14. $\frac{3\pi}{7} = \frac{6\pi}{14}$ 

Decimal Degrees vs. DMS

360 degrees (360°) describe one revolution.	
180 degrees (180°) describe $\frac{1}{2}$ of a revolution. (Notice: 180/360 = $\frac{1}{2}$.)	

So…

90° describe _____ of a revolution.

45° describe ______ of a revolution.

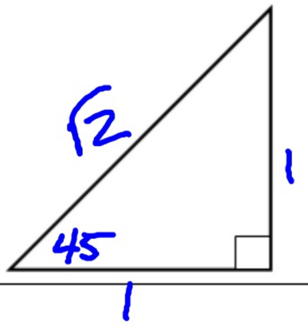
60° describe _____ of a revolution

30° describe ______ of a revolution.

1° describes ______ of a revolution.

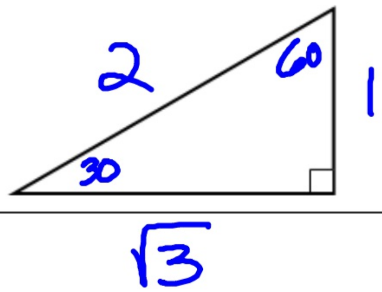
Corresponding Reference Triangles:

45°-45°-90° Triangle



30°-60°-90° Triangle

★ Note: Changing the orientation of this triangle makes either a 30° or a 60° reference angle.

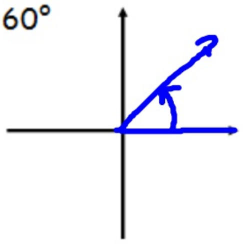
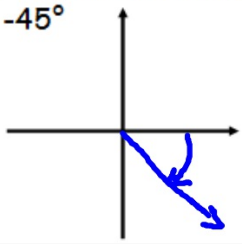
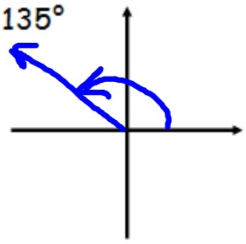
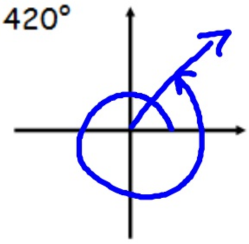


Converting between Degrees, Minutes & Seconds (D°M'S") & Decimal Degrees:

Parts of degrees are measure in **minutes** and **seconds**.

- One minute = **1'** = 1/60 of a degree.
- One second = **1"** = 1/3600 of a degree.
- Conversion Ideas: 1 counterclockwise revolution = ______. 1° = ______. 1' = _____.

Try: Draw the following angles from an initial side in standard position.

1. 60° 	2. -45° 	3. 135° 	4. 420° 

Using the Calculator for Conversions:

[ANGLE] #4: converts a decimal degree into D°M'S" form	**Example**: Convert 17.94° into D°M'S" form. Type the following into the calculator: 17.94 ▸ DMS (Press ENTER) 17°56'24"
[ANGLE] #1: gives the degree symbol [ANGLE]#2: gives the minute symbol The second symbol is above the plus-sign in green.	**Example**: Convert 124°57'8" into decimal degrees. Type the following into the calculator: 124°57'8" (Press ENTER) 124.9522222

Try: Convert the following using the graphing calculator.

7. 312°7'54" into decimal degrees	8. 187.019° into D°M'S" form
312.132°	187°1'8.4"

335: 11 → 69

e00

$$\boxed{\text{Area of Sector}}$$

$$A = \frac{1}{2} r^2 \theta$$

$\boxed{\text{Note:}}$ θ has to be in RADIANS

Area of a Sector: ____________________ θ is in **radians**!! $A = \frac{1}{2} r^2 \theta$

Ex 1 Find the area of the sector given the following conditions.

a) Find the area of the sector of a circle of radius 12 cm formed by an angle of $\frac{8\pi}{7}$ radians. $A = \frac{1}{2}(12)^2\left(\frac{8\pi}{7}\right)$ $A = \frac{576}{7}\pi \text{ cm}^2$ $A \approx 258.5081 \text{ cm}^2$
b) Find the area of the sector of a circle of radius 5 feet formed by an angle of 50°. Need to convert to RADIANS $A = \frac{1}{2}(5)^2(50°)\left(\frac{\pi}{180°}\right)$ $A = \frac{125}{36}\pi \text{ ft}^2$

$A \approx 10.908 \text{ ft}^2$

Objective: Students will be able to use the six trig ratios and apply them to the unit circle

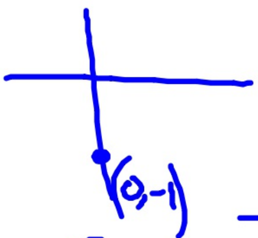
The Six Trig Ratios

From Geometry, you know three trigonometric ratios - sine, cosine, and tangent. If we let every angle be in standard position on a circle of radius r ($x^2 + y^2 = r^2$) with a point P(x, y) on the terminal side of θ,

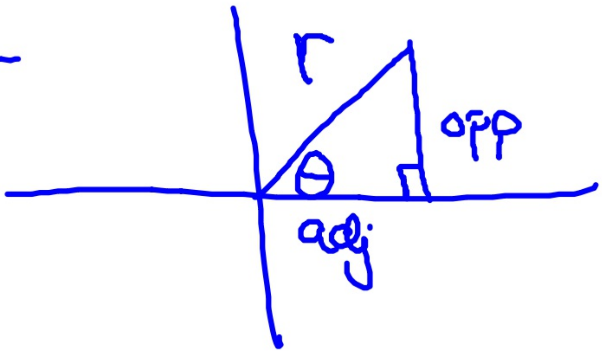
$\sin\theta = \frac{opposite}{hypotenuse} = \frac{y}{r}$	$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{x}{r}$	$\tan\theta = \frac{opposite}{adjacent} = \frac{y}{x}$

The **reciprocal trigonometric ratios** are cosecant(csc), secant(sec), and cotangent (cot).

$\csc\theta = \frac{1}{\sin\theta} = \frac{r}{y}$	$\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$	$\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$



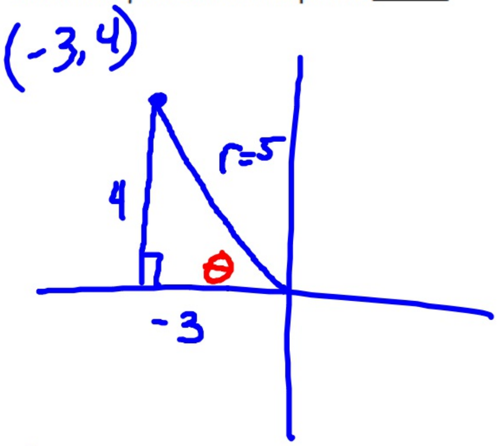
r = 1



x = 0
y = -1
r = 1

Ex 1 Find the values of the six trigonometric ratios of angle θ, if (-3, 4) is a point on its terminal side.

In which quadrant is the point? ______ What is the value of the hypotenuse? _____



$\sin\theta = 4/5$ $\csc\theta = 5/4$

$\cos\theta = -3/5$ $\sec\theta = -5/3$

$\tan\theta = 4/-3$ $\cot\theta = -3/4$

① Find r

$(-3)^2 + (4)^2 = r^2$

$9 + 16 = r^2$

$25 = r^2$

$\sqrt{25} = r$

$5 = r$

Never reduce !!

RADIAN MODE

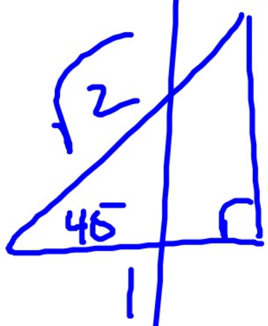
$\tan(45)$ $1.6\ldots$ | $\tan\left(\frac{\pi}{2}\right)$ 1

$\tan(45^\circ)$ 1

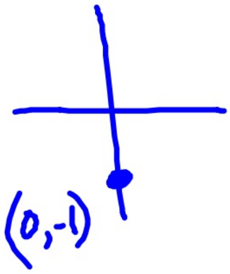
Degree

$\tan 45$ 1 | $\tan\left(\frac{\pi}{2}\right)$

$\tan 45^\circ$ 1



Ex 2 Let t be a real number and P(0, -1) be a point on the unit circle that corresponds to t. What are the values of the six trig functions?



$\sin^{-1}$

$\boxed{\sin}$

1/sin

$1/(\tan(3\pi/2))$

$1/(\tan(270))$

$\frac{\sin}{\cos}$

sin t = -1

cos t = $\frac{0}{1} = 0$

tan t = undefined

csc t = -1

sec t = undefined

cot t = 0

What is the angle in this problem?

270°

$\frac{3\pi}{2}$

$\sin t = \frac{y}{r}$

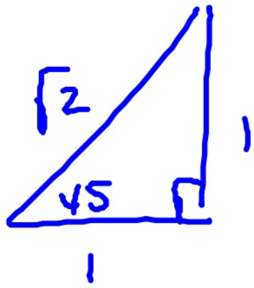
$1/\sin(270)$

$\frac{\cos(270)}{\sin(270)}$

Ex 3 Evaluate. $\tan\frac{\pi}{4} - \sin\frac{3\pi}{2}$

$1 - (-1)$

2

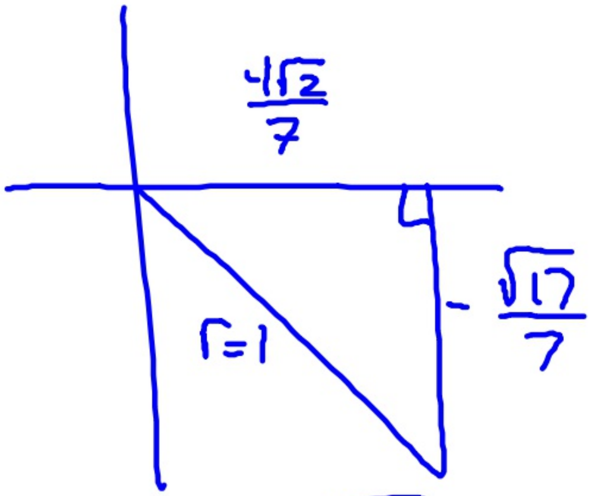


sin y

Ex 4 Let t be a real number and P = $\left(\frac{4\sqrt{2}}{7}, -\frac{\sqrt{17}}{7}\right)$ be the point on the unit circle that corresponds to t. Find the values of the six trig functions.

x y

c s



$$\left(\frac{4\sqrt{2}}{7}\right)^2 + \left(-\frac{\sqrt{17}}{7}\right)^2 = r^2$$

$$\frac{32}{49} + \frac{17}{49} = r^2$$

$$\frac{49}{49} = r^2$$

$$1 = r^2$$

$$1 = r$$

$$\sin\theta = \frac{-\sqrt{17}}{7}$$

$$\csc\theta = \frac{-7}{\sqrt{17}} = \frac{-7\sqrt{17}}{17}$$

$$\cos\theta = \frac{4\sqrt{2}}{7}$$

$$\sec\theta = \frac{7}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{8}$$

$$\tan\theta = \frac{\frac{-\sqrt{17}}{7}}{\frac{4\sqrt{2}}{7}} = \frac{-\sqrt{17}}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{34}}{8}$$

$$\cot\theta = \frac{-8}{\sqrt{34}} = \frac{-8\sqrt{34}}{34}$$

$$(2\sqrt{3})^2 = 12 \qquad (-4\sqrt{6})^2 = 96$$

$$(5\sqrt{2})^2 = 50$$

Ex 4 Let t be a real number and P = $\left(\frac{4\sqrt{2}}{7}, -\frac{\sqrt{17}}{7}\right)$ be the point on the unit circle that corresponds to t. Find the values of the six trig functions.

$\beta \qquad \alpha$

θ

$$\frac{\sqrt{14}}{7}$$

$$\frac{\frac{\sqrt{7}}{3}}{\frac{\sqrt{2}}{3}} = \frac{\sqrt{7}}{\sqrt{2}} = \frac{\sqrt{14}}{2} \qquad \frac{7}{\sqrt{14}} \cdot \frac{\sqrt{14}}{\sqrt{14}} = \frac{7\sqrt{14}}{14}$$

Answers to 5-1 Coterminal Angles (ID: 1)

1) $45°$ and $-675°$

2) $405°$ and $-315°$

3) $450°$ and $-270°$

5) $\frac{5\pi}{3}$ and $-\frac{7\pi}{3}$

6) $\frac{4\pi}{3}$ and $-\frac{2\pi}{3}$

7) $\frac{13\pi}{12}$ and $-\frac{11\pi}{12}$

9) No

10) Yes

11) Yes

13) Yes

14) No

15) Yes

17) $272°$ and $-88°$

18) $95°$ and $-625°$

19) $300°$ and $-420°$

21) $\frac{83\pi}{90}$ and $-\frac{277\pi}{90}$

22) $\frac{5\pi}{18}$ and $-\frac{31\pi}{18}$

23) $\frac{133\pi}{36}$ and $-\frac{11\pi}{36}$

22 $\frac{-103\pi}{18}$

$\frac{77\pi}{18}$

4) $270°$ and $-90°$

8) $\frac{47\pi}{12}$ and $-\frac{\pi}{12}$

12) Yes

16) No

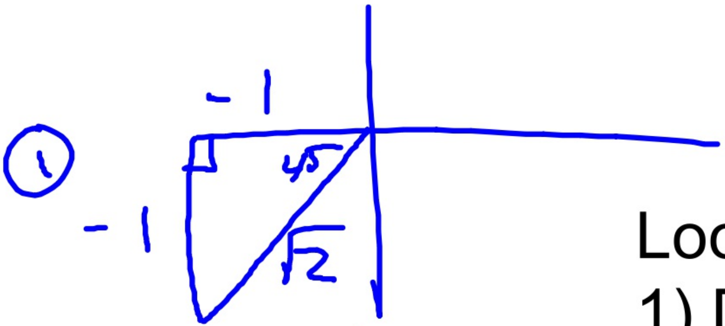
20) $318°$ and $-42°$

24) $\frac{71\pi}{18}$ and $-\frac{\pi}{18}$

Do without using a Calculator!!

3 $\csc 225$

ref $\angle = 45$



$\sin 225 = \frac{-1}{\sqrt{2}}$

2 $\csc 225 = -\sqrt{2}$

Look for:

1) Draw ref triangle

2) Correct value for trig function asked for

Answers to 5-2 Trig Functions of Any Angle (ID: 1)

1) $\frac{\sqrt{3}}{3}$

2) -1

3) $-\sqrt{2}$

4) Undefined

5) -2

6) $\frac{\sqrt{2}}{2}$

7) $\frac{1}{2}$

8) $\frac{\sqrt{3}}{2}$

9) 1

10) 2

11) 1

12) 0

13) $\frac{\sqrt{3}}{2}$

14) $\frac{\sqrt{3}}{3}$

15) $-\frac{\sqrt{3}}{2}$

16) $\sqrt{2}$

17) 2

18) $-\frac{1}{2}$

19) Undefined

20) Undefined

21) 1

22) 1

23) $-\frac{1}{2}$

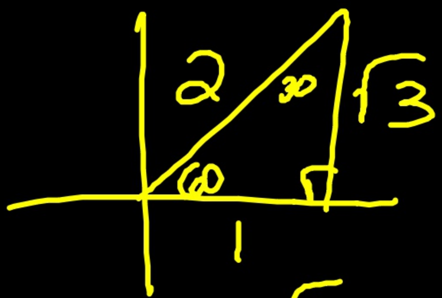
24) 0

25) $\frac{2\sqrt{3}}{3}$

26) 1

cot 60

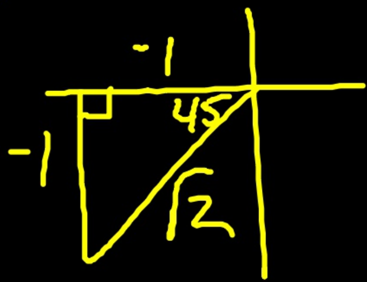
①



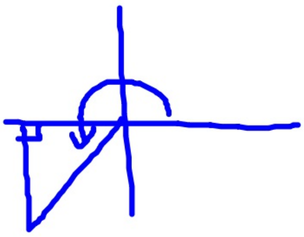
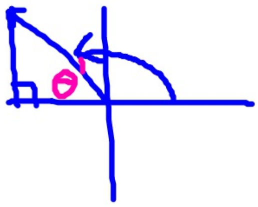
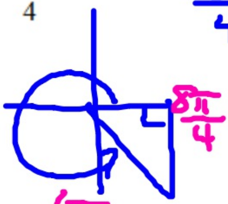
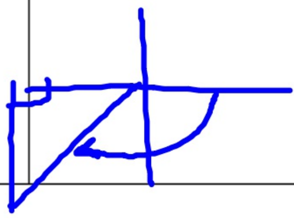
$\tan 60 = \frac{\sqrt{3}}{1}$

② $\cot 60 = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

③ $\csc 225 = -\sqrt{2}$



Ex 1 Find the reference angle, θ', given a value for θ.

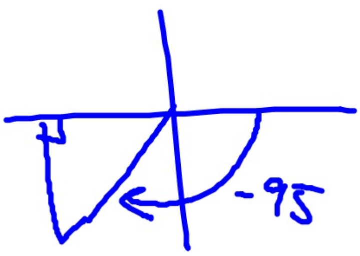
a) $250°$ -180 $\theta' = 70$ 	b) $160°$ $180 - 160 = 20$  $\theta' = 20°$
c) $\frac{7\pi}{4}$ $\frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$  $\theta' = \frac{\pi}{4}$	d) $-\frac{5\pi}{6}$ $\frac{6\pi}{6} - \frac{5\pi}{6} = \frac{\pi}{6}$  $\theta' = \pi/6$

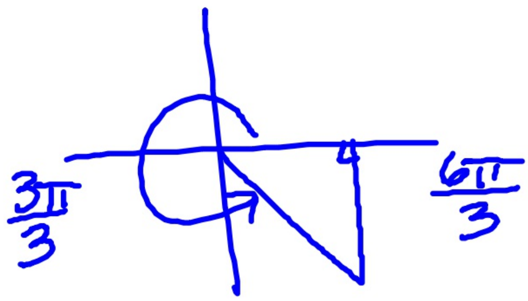
$\frac{6\pi}{4}$

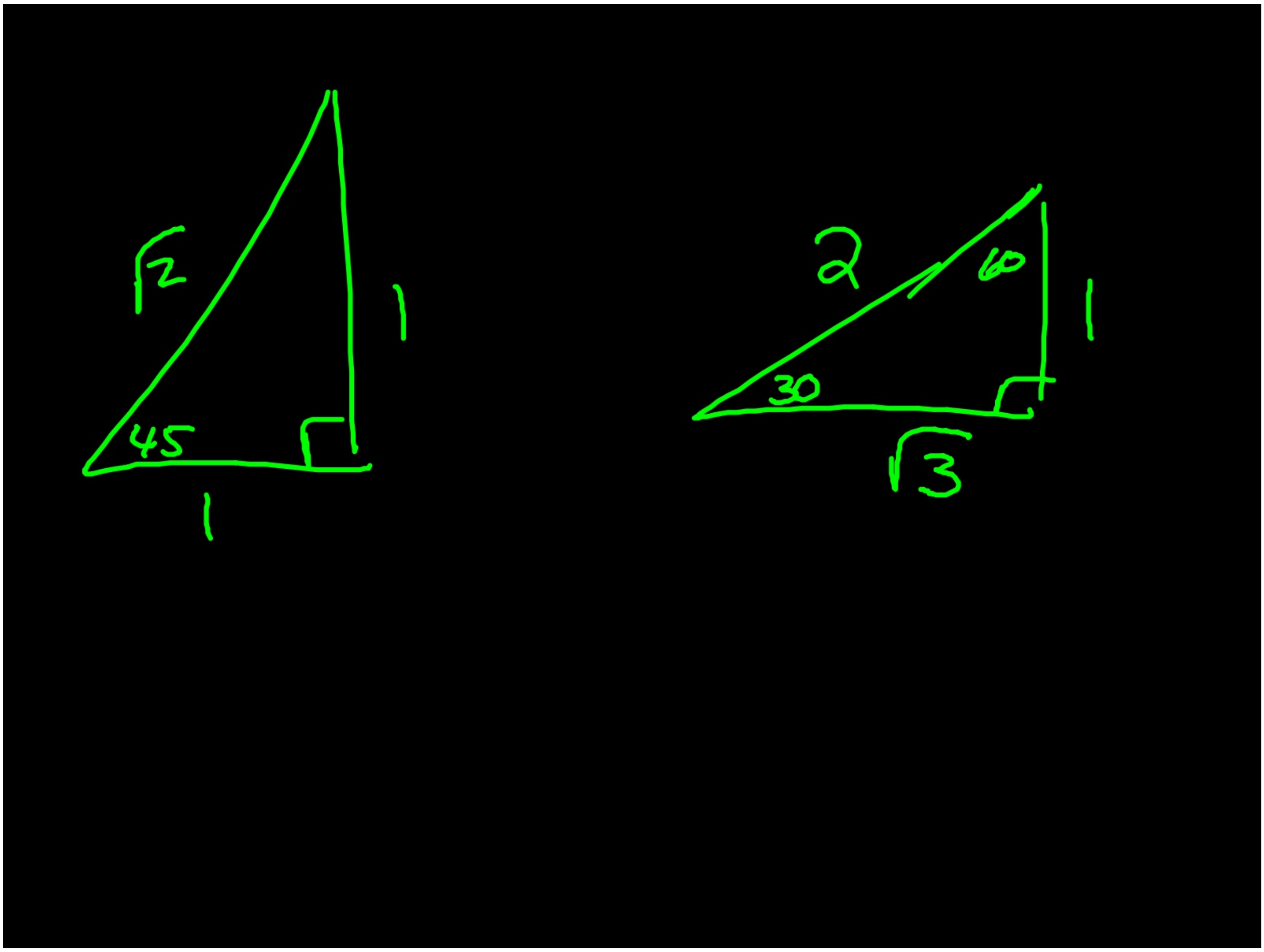
$2\pi = \frac{8\pi}{4}$

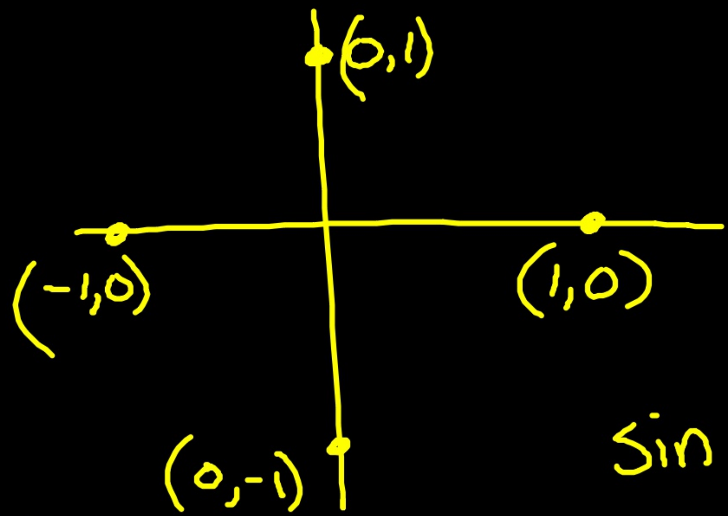
Try: Find the reference angle, θ', given a value for θ.

1. -95° $180 - 95 = 85^\circ$ $\theta' = 85^\circ$	2. $\frac{5\pi}{3}$ $\frac{6\pi}{3} - \frac{5\pi}{3} = \frac{\pi}{3}$ $\theta' = \pi/3$






√2
1
45
1
2
60
1
30
√3



$x = 1$

$y = 0$

$r = 1$

$\sin 360 = 0$ $\csc 360 =$

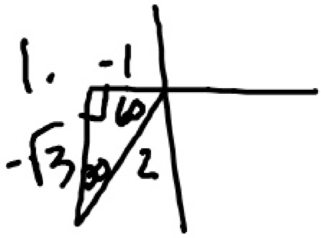
$\cos 360 = 1$

$\tan 360 = 0$

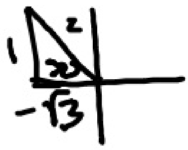
$\boxed{\begin{array}{l} \cos = x \\ \sin = y \\ r = 1 \end{array}}$

$\tan = \frac{y}{x}$

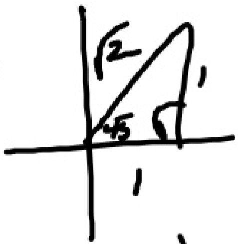
Practice Quiz Key

1. 

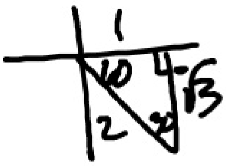
$\sec 240^\circ = -2$

2. 

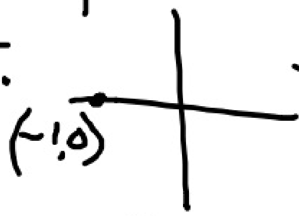
$-\dfrac{\sqrt{3}}{3}$

3. 

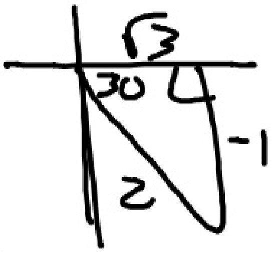
$\sqrt{2}$

4. 

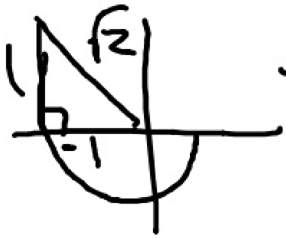
$-\dfrac{\sqrt{3}}{2}$

5. 

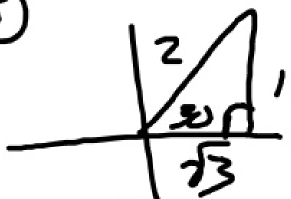
-1

6. 

$-\sqrt{3}$

7. 

$-\dfrac{\sqrt{2}}{2}$

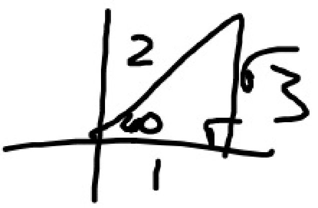
8. 

$\dfrac{2}{\sqrt{3}} \rightarrow \dfrac{2\sqrt{3}}{3}$

9. $\emptyset$

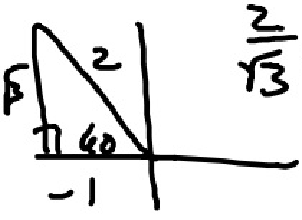
10. 

$\dfrac{\sqrt{2}}{2}$

11. 

$\dfrac{\sqrt{3}}{3}$

12. 0

13. 

$\dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

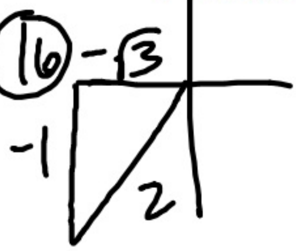
14. 

$-\dfrac{\sqrt{2}}{2}$

Practice Quiz Key

15

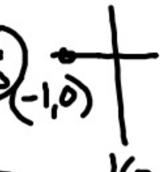

$\frac{\sqrt{2}}{2}$

16


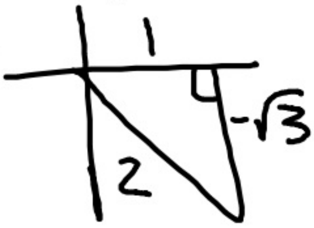
$-\frac{2\sqrt{3}}{3}$

17

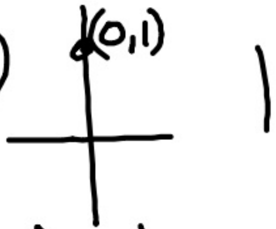

$-\sqrt{3}$

18


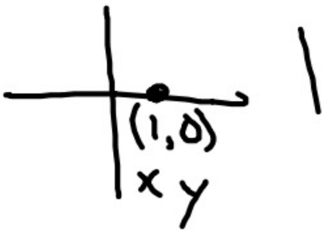
und.

19


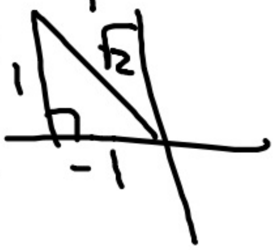
$\frac{2}{-\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

20


1

21


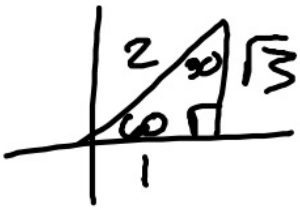
1

22


$\frac{4\pi}{4}$

-1

23 undefined

24


$\frac{1}{2}$

25

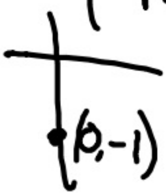

$-\frac{\sqrt{3}}{3}$

26


$\sqrt{2}$

27

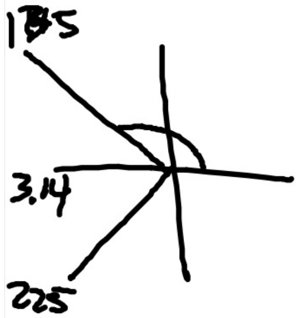

$-\frac{1}{2}$

28


$\frac{-1}{0}$ = und.

Signs of Trig Functions: (See p.360 for separate representations of the signs of trig functions.)

Sin / csc	A Tan / cot	Cos / sec	Ex 2 Find the value(s) of θ for the following equations for $0 \le \theta < 2\pi$.		
	a) $\cos\theta = -0.8471$ $\theta_1 = \cos^{-1}(-0.8471)$	b) $\tan\theta = 3.9283$	c) $\csc\theta = 4.2706$



$\theta_1 = 2.581$

$\theta' = \pi - \theta_1$

$\theta' = 3.14 - 2.581$

$\theta' = 0.559$

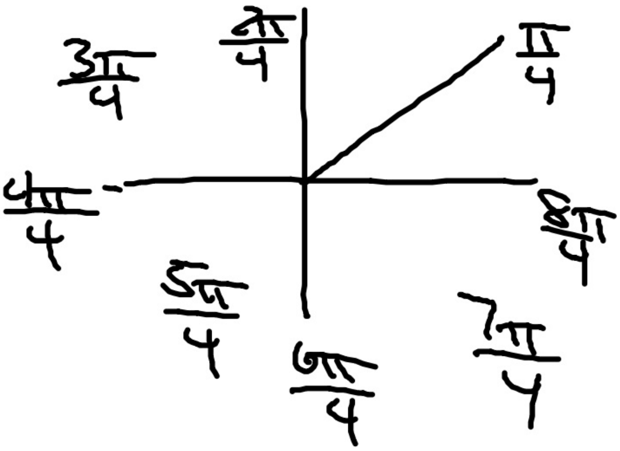
$\theta_2 = \pi + \theta'$

Ex 1 Find the exact value of the following expressions.

a) $\cos 13\pi$ SAME AS	b) $\cos\frac{15\pi}{4} =$ $\cos \frac{12\pi}{4} + \frac{3\pi}{4}$	c) $\tan\frac{9\pi}{2}$	d) $\sin\frac{19\pi}{3}$

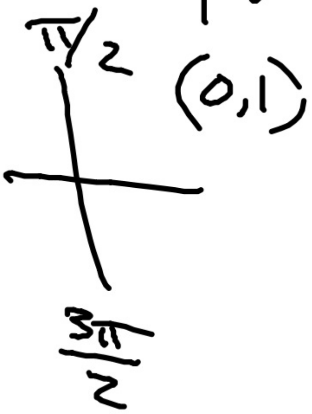
$\cos \pi$

$\cos \frac{15\pi}{4} - \frac{8\pi}{4} \rightarrow 2\pi = 1$



1E13 $\infty \rightarrow$ undef.

$1 \cdot 10^{13} = 10000000000000$



Ex 3 Find the exact values of the following expressions.

a) $\tan 40^\circ - \dfrac{\sin 40^\circ}{\cos 40^\circ}$	b) $\cos^2 \dfrac{\pi}{10} + \dfrac{1}{\csc^2 \frac{\pi}{10}}$

$$\frac{\sin 40}{\cos 40} - \frac{\sin 40}{\cos 40}$$

$$0$$

$$\tan 40 - \tan 40$$

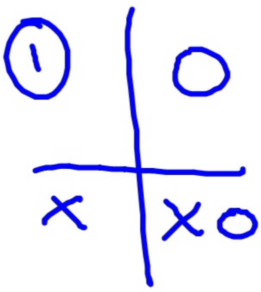
$$0$$

$$\cos^2\left(\frac{\pi}{10}\right) + \sin^2\left(\frac{\pi}{10}\right)$$

$$1$$

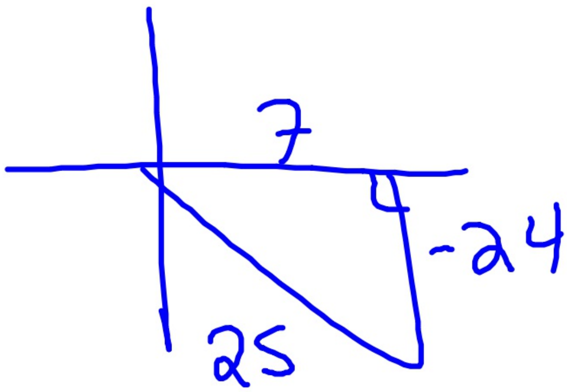
Name the quadrant in which angle θ lies.

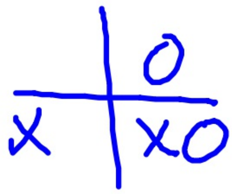
1. $\sin\theta < 0$, $\cos\theta > 0$ x O IV	2. $\sin\theta > 0$, $\cos\theta < 0$	3. $\sec\theta < 0$, $\tan\theta > 0$
4. $\cos\theta < 0$, $\csc\theta < 0$	5. $\sec\theta > 0$, $\cot\theta > 0$	6. $\tan\theta < 0$, $\sin\theta > 0$



Given sin θ and cos θ, find the *exa*

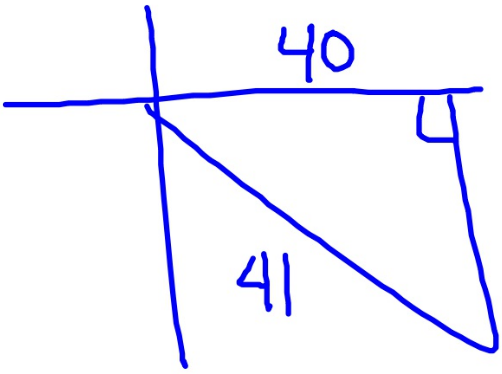
7. $\sin\theta = -\dfrac{24}{25}$ and $\cos\theta = \dfrac{7}{25}$





Find the *exact* values of each trigonometri

11. $\cos\theta = \frac{40}{41}$ and θ is in quadrant IV



$$x^2 + y^2 = r^2$$

$$40^2 + y^2 = 41^2$$

$$1600 + y^2 = 1681$$

$$y^2 = 81$$

$$y = \pm\sqrt{81}$$

$$\boxed{y = -9}$$