Pre-Calculus

Functions Review (Chapters 1, 2 & 4)

1. Find the average rate of change of $f(x) = x^2 + 3$ from 1 to 5.

$f(5) = (5)^2 + 3 = 28$

$f(1) = (1)^2 + 3 = 4$

$\frac{f(5) - f(1)}{5 - 1} = \frac{28 - 4}{4} = 6$

2. Given

$$h(x) = \begin{cases} 45, & \text{if } 0 \le x < 225 \\ 0.13x + 30, & \text{if } x \ge 225 \end{cases}$$

find $h(248)$.

$h(248) = 0.13(248) + 30$

$\boxed{h(248) = 62.24}$

3. If $j(x) = \frac{3x}{x^2 + 4}$, find

a) $j(-7)$ a)__________

$j(-7) = \frac{7(-3)}{(-7)^2 + 4} = \frac{-21}{51} = -\frac{7}{17}$

b) $j(2x)$ b)__________

$j(2x) = \frac{3(2x)}{(2x)^2 + 4} = \frac{6x}{4x^2 + 4}$

$j(2x) = \frac{6x}{2(2x^2 + 2)} = \frac{3x}{2x^2 + 2}$

If $f(x) = 2x^2 - 3x + 5$ and $g(x) = x + 3$, find simplified expressions for the following.

4. $f(x) + 7$

$(2x^2 - 3x + 5) + 7$

$\boxed{2x^2 - 3x + 12}$

5. $f(x + 2)$

$2(x+2)^2 - 3(x+2) + 5$

$2(x^2 + 4x + 4) - 3x - 6 + 5$

$2x^2 + 8x + 8 - 3x - 1$

$\boxed{2x^2 + 5x + 7}$

6. $-f(x)$

$-(2x^2 - 3x + 5)$

$\boxed{-2x^2 + 3x - 5}$

7. g^{-1}

$y = x + 3$

$x = y + 3$

$x - 3 = y$

$\boxed{g^{-1}(x) = x - 3}$

8. $f \circ g = f(g(x))$

$= 2(x+3)^2 - 3(x+3) + 5$

$= 2(x^2 + 6x + 9) - 3x - 9 + 5$

$= 2x^2 + 12x + 18 - 3x - 4$

$= \boxed{2x^2 + 9x + 14}$

9. $g \circ f = g(f(x))$

$= (2x^2 - 3x + 5) + 3$

$= \boxed{2x^2 - 3x + 8}$

10. the difference quotient, $\frac{f(x+h)-f(x)}{h}$

$$\frac{1}{h}\left[\left[2(x+h)^2-3(x+h)+5\right]-\left[2x^2-3x+5\right]\right]$$

$$\frac{1}{h}\left[2(x^2+2xh+h^2)-3x-3h+5\right]-\left[2x^2-3x+5\right]$$

$$\frac{1}{h}\left[\cancel{2x^2}+4xh+2h^2-\cancel{3x}-3h+\cancel{5}-\cancel{2x^2}+\cancel{3x}-\cancel{5}\right]$$

$$\frac{1}{h}\left[4xh+2h^2-3h\right]$$

$$\frac{4xh}{h}+\frac{2h^2}{h}-\frac{3h}{h} \Longrightarrow 4x+h-3 \Rightarrow \boxed{4x-3+h}$$

Determine if the following functions are *even*, *odd*, or *neither* algebraically.

11. $f(x) = 4x^3 - 5x + 2$

$f(-x) = 4(-x)^3 - 5(-x) + 2$

$f(-x) = -4x^3 + 5x + 2$

$-f(x) = -4x^3 + 5x - 2$

Since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, $f(x)$ is neither an even function nor an odd function.

12. $g(x) = \frac{2x}{x^2+3}$

$g(-x) = \frac{2(-x)}{(-x)^2+3} = \frac{-2x}{x^2+3}$

$-g(x) = \frac{-2x}{x^2+3}$

Since $g(-x) = -g(x)$, $g(x)$ is an odd function.

If $f(x) = \frac{5x+4}{x-8}$ and $g(x) = \frac{2}{x+1}$, find the following. Please state the domain of each.

13. f^{-1}

$y = \frac{5x+4}{x-8}$

$x = \frac{5y+4}{y-8}$

$xy - 8x = 5y + 4$

$xy - 5y = 8x + 4$

$y(x-5) = 8x + 4$

$y = \frac{8x+4}{x-5}$

$f^{-1}(x) = \frac{8x+4}{x-5}$, $\{x \mid x \neq 5\}$

$(-\infty, 5) \cup (5, \infty)$

14. $f \circ g \rightarrow f(g(x)) \rightarrow$ since g is "inside function", $x \neq -1$

$$(f\circ g)(x) = \frac{5\left(\frac{2}{x+1}\right)+4}{\frac{2}{x+1}-8} = \frac{\frac{10}{x+1}+\frac{4(x+1)}{(x+1)}}{\frac{2}{x+1}-\frac{8(x+1)}{x+1}}$$

$$= \frac{\frac{10+4x+1}{x+1}}{\frac{2-8x-8}{x+1}} = \frac{\frac{4x+11}{x+1}}{\frac{-8x-6}{x+1}} = \frac{4x+11}{-8x-6}$$

$(f\circ g)(x) = \frac{4x+11}{-8x-6}$

$-8x-6 \neq 0$

$-8x \neq 6$

$x \neq \frac{-3}{4}$

Domain: $\{x \mid x \neq -3/4 \text{ AND } x \neq -1\}$

—OR— $(-\infty, -3/4) \cup (-3/4, -1) \cup (-1, \infty)$

10 $f(x) = 2x^2 - 3x + 5$ find $\dfrac{f(x+h) - f(x)}{h}$

$$\frac{[2(x+h)^2 - 3(x+h) + 5] - [2x^2 - 3x + 5]}{h}$$

$$\frac{[2(x^2 + 2xh + h^2) - 3x - 3h + 5] - [2x^2 - 3x + 5]}{h}$$

$$\frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 5 - 2x^2 + 3x - 5}{h}$$

$$\frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{3x} - 3h + \cancel{5} - \cancel{2x^2} + \cancel{3x} - \cancel{5}}{h}$$

$$\frac{4xh + 2h^2 - 3h}{h}$$

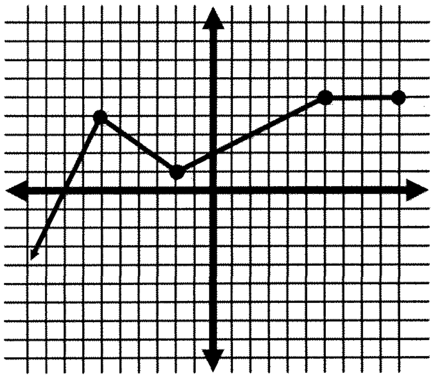
$$\frac{4xh}{h} + \frac{2h^2}{h} - \frac{3h}{h}$$

$$4x + 2h - 3$$

$$\boxed{4x - 3 + 2h}$$

You will see this again in CALCULUS... WOOT WOOT!

19. Given the graph of $h(x)$, find the following:



a) the domain of $h(x)$ $(-\infty, 10]$

b) the range of $h(x)$ $(-\infty, 5]$

c) $h(7)$ $\varnothing$ or undefined or DNE

d) the value(s) of x for which $h(x) = -2$
$x = -9$ or $h(-9) = -2$

e) the number of times the graph of $y = 3$ intersects the graph of $h(x)$
3

f) the x-intercept(s) $(-8, 0)$

g) the y-intercept $(0, 2)$

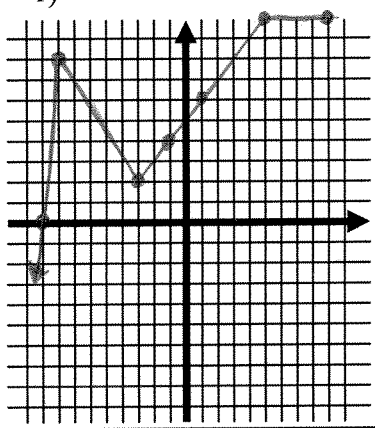
h) the increasing intervals
$(-\infty, -6) \cup (-2, 6)$

i) the decreasing intervals $(-6, -2)$

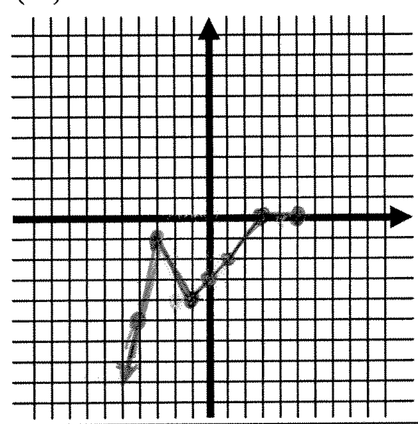
j) the intervals when $h(x) > 0$
$(-8, 10)$

k) the local maxima $(-6, 4)$

l) the graph of $2 \cdot h(x + 1)$



m) the graph of $h(2x) - 5$



$\boxed{h(x)}$

x	y
-8	0
-6	4
-2	1
0	2
2	3
6	5
10	5

$\boxed{h(x+1)}$

x	y
-9	0
-7	4
-3	1
-1	2
1	3
5	5
9	5

move Left 1 unit

$\boxed{2h(x+1)}$

x	y
-9	0
-7	8
-3	2
-1	4
1	6
5	10
9	10

Double y-coordinate

$\boxed{h(x)}$

x	y
-8	0
-6	4
-2	1
0	2
2	3
6	5
10	5

$\boxed{h(2x)}$

x	y
-4	0
-3	4
-1	1
0	2
1	3
3	5
5	5

Divide EACH X by 2

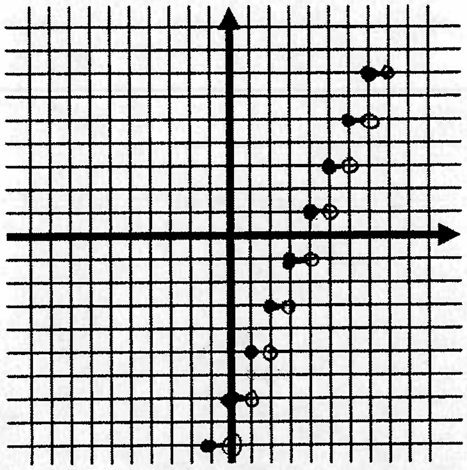
$\boxed{h(2x)-5}$

x	y
-4	-5
-3	-1
-1	-4
0	-3
1	-2
3	0
5	0

Decrease each y by 5

Graph the following functions. State the domain and range.

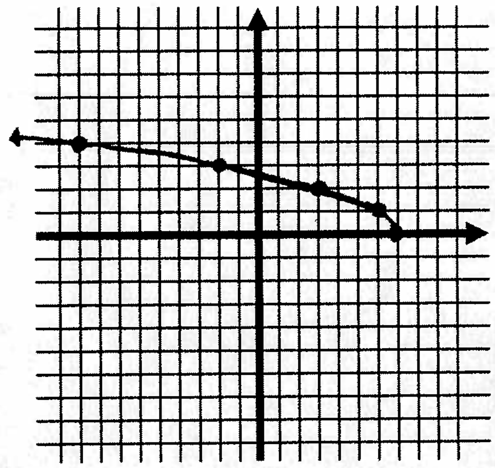
15. $f(x) = 2\text{int}(x) - 7$



Domain: $(-\infty, \infty)$

16. $g(x) = \sqrt{7-x}$

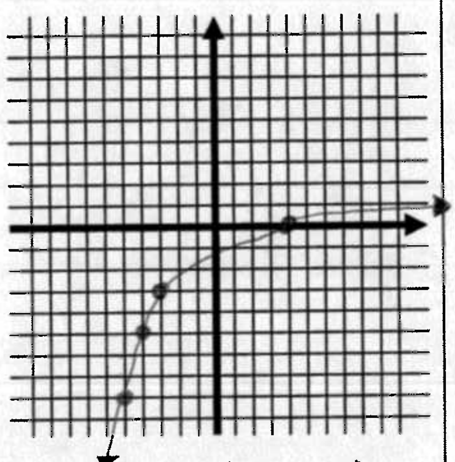
x	g(x)
7	0
6	1
3	2
-2	3
-9	4



Domain: $(-\infty, 7]$ Range: $[0, \infty)$

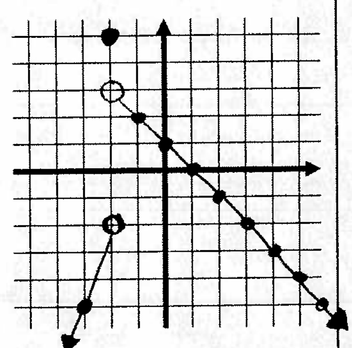
Graph the following functions. State the domain and range.

17. $f(x) = 3\sqrt[3]{x+4} - 6$



Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

18. $g(x) = \begin{cases} 3x+4, & x < -2 \\ 5, & x = -2 \\ -x+1, & x > -2 \end{cases}$



Domain: $(-\infty, \infty)$ Range: $(-\infty, 3) \cup (5)$