Matrix - a rectangular arrangement of numbers in rows and columns

Dimensions - The dimensions of a matrix with m rows and n columns are m x n.

Elements (entries) - the numbers in a matrix

Equal Matrices - Two matrices are equal if they have the same dimensions and equal elements in corresponding positions.

Scalar: a number by which you multiply a matrix

Scalar Multiplication: The process of multiplying each element by a scalar

* To add or subtract matrices, add or subtract corresponding elements

* To multiply by a scalar, multiply all elements by the scalar

Ex 1. Simplify the following matrices.

a) $\begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$	b) $\begin{bmatrix} 4 & 6 \\ -2 & 2 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -4 & 2 \\ -3 & 1 \end{bmatrix}$	c) $4\begin{bmatrix} 7 & -1 & 4 & 3 \\ 0 & 8 & -9 & 6 \end{bmatrix}$	d) $-3\begin{bmatrix} -1 & 5 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 5 & -3 \end{bmatrix}$
$\begin{bmatrix} -1 & 6 \\ 4 & 1 \end{bmatrix}$	$\begin{bmatrix} 2 & 3 \\ 2 & 0 \\ 4 & 4 \end{bmatrix}$	$\begin{bmatrix} 28 & -4 & 16 & 12 \\ 0 & 32 & -36 & 24 \end{bmatrix}$	$\begin{bmatrix} 1 & -14 \\ -7 & -3 \end{bmatrix}$

Ex. 2 A local bakery keeps track of their sales as shown below. Organize that data using matrices, then write and interpret a matrix giving the total bakery items sold per store.

Last Month:	**This Month**:
Store 1: 650 rolls, 220 cakes, 32 pies;	Store 1: 840 rolls, 250 cakes, 50 pies;
Store 2: 540 rolls, 200 cakes, 30 pies	Store 2: 800 rolls, 250 cakes, 42 pies

$$\begin{bmatrix} 650 & 220 & 32 \\ 540 & 200 & 30 \end{bmatrix} + \begin{bmatrix} 840 & 250 & 50 \\ 800 & 250 & 42 \end{bmatrix}$$

$$\begin{bmatrix} 1490 & 470 & 82 \\ 1340 & 450 & 72 \end{bmatrix}$$

Ex 3 The following matrix represents the inventory of a chain of entertainment stores. If CDs cost $15, DVDs cost $20, VHSs cost $18, and Games Cost $30, what is the total value of the inventory at each store?

$$\begin{array}{c} \\ Store\ 1 \\ Store2 \\ Store\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} CDs & DVDs & VHSs & Games \end{array} \\ \begin{bmatrix} 2800 & 550 & 200 & 150 \\ 2600 & 800 & 150 & 120 \\ 1850 & 650 & 190 & 100 \end{bmatrix} \end{array} \qquad \begin{array}{c} CDs \\ DVDs \\ VHSs \\ Games \end{array}\begin{bmatrix} 15 \\ 20 \\ 18 \\ 30 \end{bmatrix}$$

Store 1 had an inventory value of $61,100.

Store 2 had an inventory value of $61,300.

Store 3 had an inventory value of $47,170

Ex 6 Given the following matrices, evaluate the following.

$$A = \begin{bmatrix} 3 & -2 \\ 0 & 4 \\ -1 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -3 \\ 1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$$

a) A(B - C)

$$\begin{bmatrix} -10 & -16 \\ 20 & 8 \\ 25 & 14 \end{bmatrix}$$

b) AB - AC

$$\begin{bmatrix} -10 & -16 \\ 20 & 8 \\ 25 & 14 \end{bmatrix}$$

Does A(B - C) = AB - AC? Is the distributive property valid for matrices?

To use a calculator for matrices:

1. Use the MATRIX button.

   To enter a matrix: MATRIX → EDIT

   Choose the letter you want to name the matrix.

   Type in the dimensions (rows x columns)

   Type in the elements (Press ENTER between each element.)

   Press 2nd QUIT to exit.

2. To do operations with matrices:

   MATRIX → NAMES

   Choose the matrix you want to go first.

   Type the operation you want to perform.

   Repeat steps 1-3 until you have the expression you're evaluating.

   Press ENTER to get the sum, difference or product.

Multiplying matrices together:

You can multiply two matrices together if the number of columns in matrix A equals the number of rows in matrix B.

A · B = AB

(m x n) (n x p) = (m x p)

Ex 4 Is AB defined or undefined? What are the dimensions of the product, if possible?

Matrix A's dim.	Matrix B's dim.	Defined or not	Dimensions
a) 2 x 4	4 x 3	Defined	2 x 3
b) 5 x 7	7 x 2	Defined	5 x 2
c) 10 x 2	2 x 13		10 x 13
d) 1 x 4	1 x 4	Not Defined	

Ex 5 Given matrix A and matrix B, find the following products. $A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$ $\begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$

a) AB

$$\begin{bmatrix} 2\cdot 1 + -3\cdot 3 & 2\cdot -4 + -3\cdot -2 \\ 1\cdot 1 + 5\cdot 3 & 1\cdot -4 + 5\cdot -2 \end{bmatrix}$$

$$\begin{bmatrix} -7 & -2 \\ 16 & -14 \end{bmatrix}$$

b) BA

$$\begin{bmatrix} 1\cdot 2 + -4\cdot 1 & 1\cdot -3 + -4\cdot 5 \\ 3\cdot 2 + -2\cdot 1 & 3\cdot -3 + -2\cdot 5 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -23 \\ 4 & -19 \end{bmatrix}$$

Does AB = BA? Is the multiplication of matrices commutative?

Ex 6 Given the following matrices, evaluate the following.

$$A = \begin{bmatrix} 3 & -2 \\ 0 & 4 \\ -1 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -3 \\ 1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 3 \cdot 2 + (-2)(1) \\ 0 \cdot 2 + 4(1) \\ -1 \cdot 2 + 5(1) \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 4 \\ 3 \end{bmatrix}$$

AA Objective: Students will be able to find the determinant up to a 3x3 by hand and higher sized determinants using a Calculator. Students will be able to find the inverse matrix of a given matrix both by hand and calculator. Students will be able to solve using matrices.

Day 2 Notes: Matrix Determinants and Inverse

Determinant – a number associated with every square matrix, given by det A or by $|A|$

$$|A| \qquad\qquad [A]$$

The det. of A

Matrix A

Ex 1 Find the determinants of the following square matrices.

a) $\begin{bmatrix} 6 & -2 \\ 1 & -4 \end{bmatrix}$	b) $\begin{bmatrix} 4 & -2 & 0 \\ 1 & 1 & -2 \\ 2 & 5 & 3 \end{bmatrix} \begin{matrix} 4 & -2 \\ 1 & 1 \\ 2 & 5 \end{matrix}$

$-24-(-2) = -22$

$-24+2 = -22$

$(4)(1)(3)+(-2)(-2)(2)$

Ex 1 Find the determinants of the following square matrices.

a) $\begin{bmatrix} 6 & -2 \\ 1 & -4 \end{bmatrix} = \begin{vmatrix} 6 & -2 \\ 1 & -4 \end{vmatrix}$

$= -24 - (-2)$

$= -22$

b) $\begin{bmatrix} 4 & -2 & 0 \\ 1 & 1 & -2 \\ 2 & 5 & 3 \end{bmatrix} \begin{matrix} 4 & -2 \\ 1 & 1 \\ 2 & 5 \end{matrix}$

$(12+8+0) - (0-40-6)$

$20 - (-46)$

$\boxed{66}$

Identity Matrix – a square matrix (n x n) that has 1's on the diagonal and 0's elsewhere

Inverse Matrix – Two matrices are inverses of each other if their products, AB and BA, are equal to the Identity Matrix.

Using a calculator:	**Using your brain:**
Use $\boxed{x^{-1}}$ to find the inverse of a matrix.	If $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^{-1}=\dfrac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

<u>Ex 2</u> Find the inverse of $A=\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$

$$A^{-1}=\frac{1}{\det A}\begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$$

$\det A$

$|A| = 3\cdot 4 - 2\cdot 2$

$|A| = 8$

$$A^{-1}=\frac{1}{8}\begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$$

$$A^{-1}=\begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}$$

$[A]^{-1}$

b) $\begin{bmatrix} 4 & -2 & 0 \\ 1 & 1 & -2 \\ 2 & 5 & 3 \end{bmatrix} \begin{matrix} 4 & -2 \\ 1 & 1 \\ 2 & 5 \end{matrix}$

$$\left((4)(1)(3) + (-2)(-2)(2) + (0)(1)(5) \right]$$

$$\left[(12) + (8) + 0 \right] - \left[0 + (-40) + (-6) \right]$$

$$20 \quad - \quad [-46]$$

$$\boxed{66}$$

$\boxed{\text{Identity Matrix}}$

2x2

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3x3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4x4

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

NOT IDENTITY MATRICES

Matrix Equation: A system that is converted to a matrix form.

Ex 3 Rewrite the following systems as matrix equations.

a) $-2x - 5y = -19$ $3x + 2y = 1$	b) $2x + y = -13$ $x - 3y = 11$ $\begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -13 \\ 11 \end{bmatrix}$

$$\underbrace{\begin{bmatrix} -2 & -5 \\ 3 & 2 \end{bmatrix}}_{A}\begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} -19 \\ 1 \end{bmatrix}}_{B}$$

$$AX = B$$

$$X = A^{-1}B$$

Notes Day 3 – Matrix Applications

Ex 1 The approximate coordinates (in miles) of a triangular region representing a city and its suburbs are (10, 20), (-8, 5), and (-4, -5). What is the area of this region?

$$A = \pm\frac{1}{2}\begin{vmatrix} 10 & 20 & 1 \\ -8 & 5 & 1 \\ -4 & -5 & 1 \end{vmatrix}$$

$$A = \pm\frac{1}{2}\left|\begin{array}{ccc|cc} 10 & 20 & 1 & 10 & 20 \\ -8 & 5 & 1 & -8 & 5 \\ -4 & -5 & 1 & -4 & -5 \end{array}\right.$$

$$A = \pm\frac{1}{2}\left[(50 - 80 + 40) - (-20 - 50 - 160)\right]$$

$$A = \pm\frac{1}{2}\left[10 - (-230)\right]$$

$$A = \pm\frac{1}{2}\left[240\right]$$

$$\boxed{\begin{array}{l} A = 120 \text{ square units} \\ A = 120 \text{ units}^2 \end{array}}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

HW 2

$X = A^{-1}B$

Solve the linear system using Inverse Matrices.

9. $3x - 3y = -6$

$4x + y = 22$

$$\begin{bmatrix} 3 & -3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ 22 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{15} & \frac{1}{5} \\ \frac{-4}{15} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} -6 \\ 22 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} =$$

10. $2x + 3y = 0$

$5x + 7y = -1$

Inv. of $\begin{bmatrix} 3 & -3 \\ 4 & 1 \end{bmatrix}$

$$\frac{1}{3-(-12)} \begin{bmatrix} 1 & 3 \\ -4 & 3 \end{bmatrix}$$

$$\frac{1}{15} \begin{bmatrix} 1 & 3 \\ -4 & 3 \end{bmatrix}$$

12. $3x+6y+z=3$

$x+3y+z=3$

$3x+y-2z=-5$

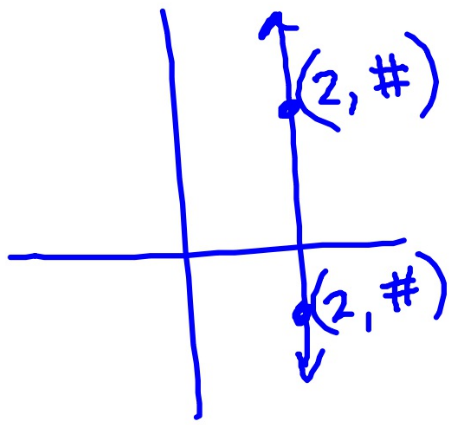
$$\begin{bmatrix} 3 & 6 & 1 \\ 1 & 3 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -5 \end{bmatrix}$$

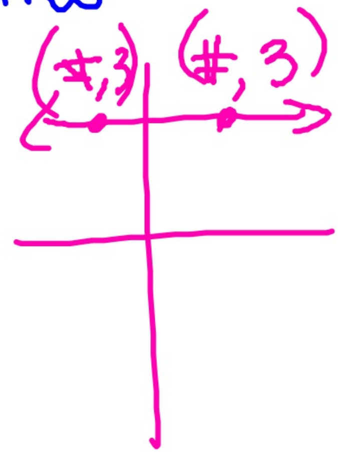
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & 6 & 1 \\ 1 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 3 \\ -5 \end{bmatrix}$$

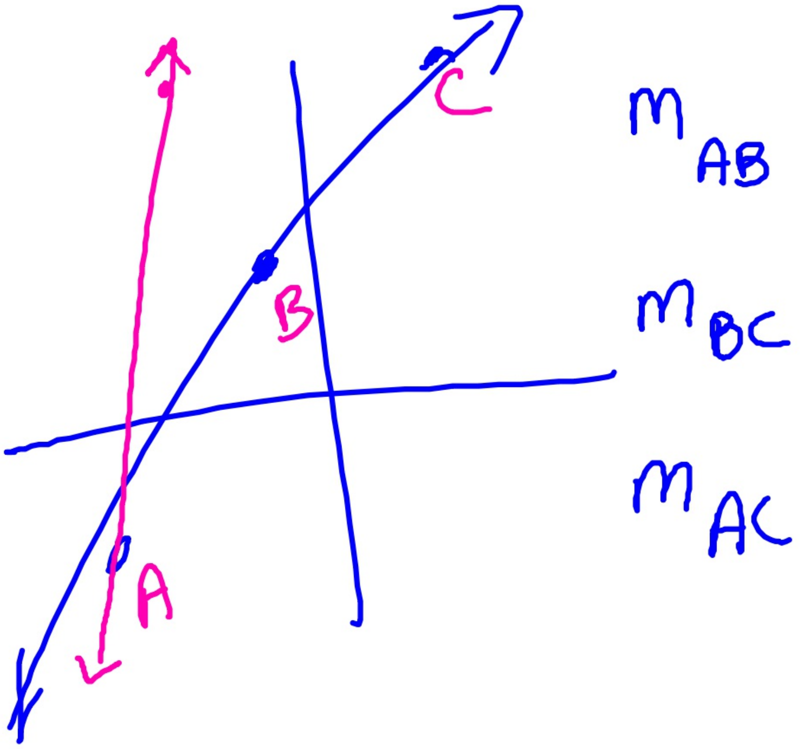
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$$

Collinear Points

on same line





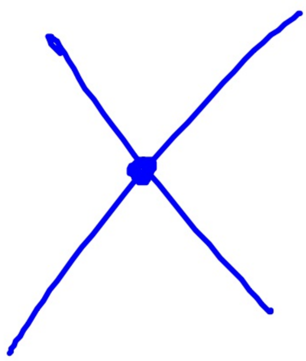


A
B
C
m_{AB}
m_{BC}
m_{AC}

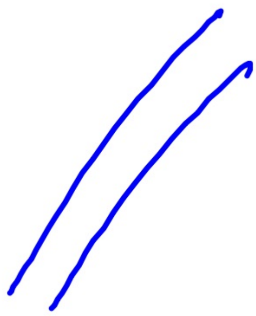
Jessica's Way – $\frac{1}{2}$ det.

#6

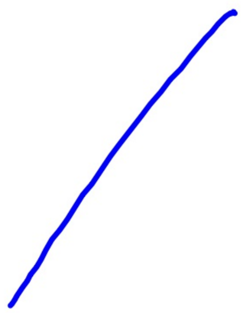
$$\frac{1}{2}\begin{vmatrix} -9 & 0 & 1 \\ -6 & -2 & 1 \\ 1 & -\frac{20}{3} & 1 \end{vmatrix} = 0$$



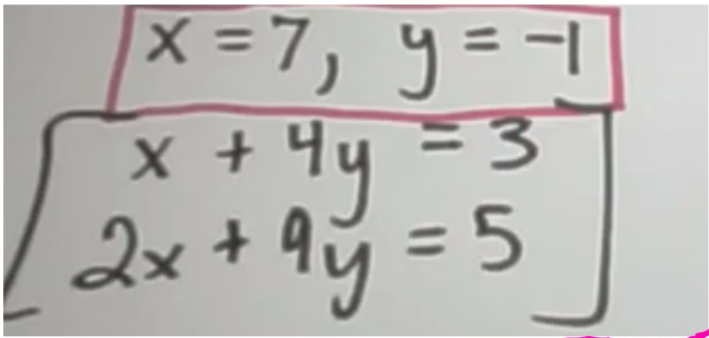
1 solution

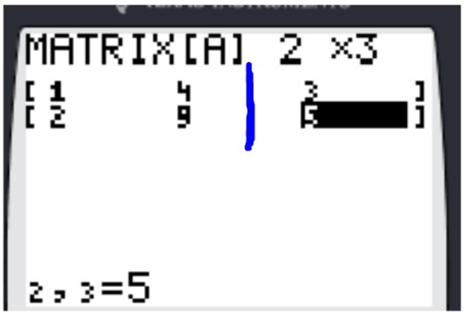


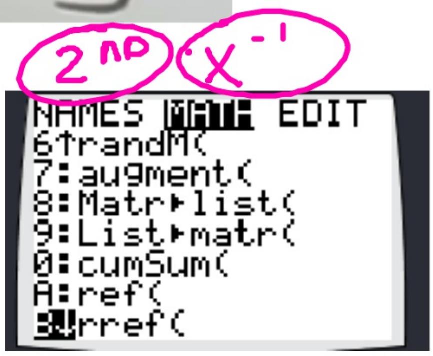
No Solution

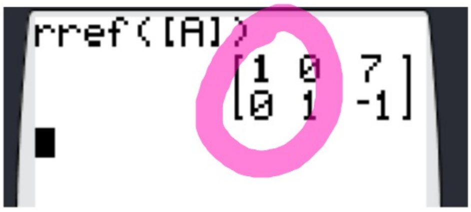


infinitely many solutions


x = 7, y = -1
x + 4y = 3
2x + 9y = 5


MATRIX[A] 2 ×3
[1 4 3]
[2 9 5]
2,3=5


2ND
X-1
NAMES MATH EDIT
6↑randM(
7:augment(
8:Matr▸list(
9:List▸matr(
0:cumSum(
A:ref(
B:rref(


rref([A])
[1 0 7]
[0 1 -1]