Section 4.6 Date_________

Logarithmic and Exponential Equations

- ✯ For logarithmic equations, use properties to condense to a single log on a side before rewriting the equation as an exponent.
- ✯ For exponential equations, isolate the base and power, take the log of both sides, and use properties of logarithms to isolate the variable.
- ✯ Check for extraneous solutions!!

<u>Ex 1: Solve the logarithmic equations.</u>

a) $-2\log_4 x = \log_4 9$

$\log_4 x^{-2} = \log_4 9$

$x^{-2} = 9$

$\frac{1}{x^2} = 9$

$x^2 = \frac{1}{9}$

$x = \pm\sqrt{\frac{1}{9}}$

$x = \pm\frac{1}{3}$

$x = -\frac{1}{3}, \frac{1}{3}$

$\log_4\left(-\frac{1}{3}\right)$ is undefined

$\therefore x = \frac{1}{3}$ is the only solution.

b) $\log_4 x + \log_4(x-3) = 1$

$\log_4\left(x(x-3)\right) = 1$

$4^1 = x(x-3)$

$4 = x^2 - 3x$

$0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$x - 4 = 0 \quad x + 1 = 0$

$x = 4 \quad or \quad x = -1$

Since $\log_4(-1)$ is undefined,

x = 4 is the solution.

c) $\ln(x+1)-\ln(x)=2$

$$\ln\left(\frac{x+1}{x}\right)=2$$

$$\frac{x+1}{x}=e^2$$

$$x+1=xe^2$$

$$1=xe^2-x$$

$$1=x(e^2-1)$$

$$\frac{1}{e^2-1}=x$$

$$0.156\approx x$$

d) $\log_2(3x+2)-\log_4 x=3$

$$\log_2(3x+2)-\frac{\log x}{\log 4}=3$$

$$\log_2(3x+2)-\frac{\log_2 x}{\log_2 4}=3$$

$$\log_2(3x+2)-\frac{\log_2 x}{2}=3$$

$$2\log_2(3x+2)-\log_2 x=6$$

$$\log_2(3x+2)^2-\log_2 x=6$$

$$\log_2\left(\frac{(3x+2)^2}{x}\right)=6$$

$$2^6=\frac{(3x+2)^2}{x}$$

$$64=\frac{(3x+2)^2}{x}$$

$$64x=(3x+2)^2$$

$$64x=9x^2+12x+4$$

$$9x^2-52x+4=0$$

$$x=\frac{(52)\pm\sqrt{(-52)^2-4(9)(4)}}{2(9)}$$

$$x=\frac{52\pm\sqrt{2560}}{18}$$

$$x=\frac{52-\sqrt{2560}}{18} \quad or \quad x=\frac{52+\sqrt{2560}}{18}$$

$$x\approx 0.078 \quad or \quad x\approx 5.700$$

e) $\log_2 x^{\log_2 x}=4$

$$\log_2 x \bullet \log_2 x=4$$

$$(\log_2 x)^2=4$$

$$\log_2 x=\pm\sqrt{4}$$

$$\log_2 x=\pm 2$$

$$\log_2 x=-2 \quad or \quad \log_2 x=2$$

$$2^{-2}=x \quad or \quad 2^2=x$$

$$\frac{1}{4}=x \quad or \quad 4=x$$

Ex 2: Solve the exponential equations.

a)

$$2^{2x}+2^x-12=0$$
$$\left(2^x\right)^2+2^x-12=0$$
$$(2^x-3)(2^x+4)=0$$
$$2^x-3=0 \quad or \quad 2^x+4=0$$
$$2^x=3 \quad or \quad 2^x=-4$$
$$\log 2^x=\log 3 \qquad \textit{No solution}$$
$$x\log 2=\log 3$$
$$x=\frac{\log 3}{\log 2}$$
$$x\approx 1.585$$

b)

$$3^x=14 \qquad \text{or} \qquad 3^x=14$$
$$\log 3^x=\log 14 \qquad \ln 3^x=\ln 14$$
$$x\log 3=\log 14 \qquad x\ln 3=\ln 14$$
$$x=\frac{\log 14}{\log 3} \qquad x=\frac{\ln 14}{\ln 3}$$
$$x\approx 2.402 \qquad x\approx 2.402$$

c)

$$2^{x+1}=5^{1-2x}$$
$$\ln 2^{x+1}=\ln 5^{1-2x}$$
$$(x+1)\ln 2=(1-2x)\ln 5$$
$$x\ln 2+\ln 2=\ln 5-2x\ln 5$$
$$x\ln 2+2x\ln 5=\ln 5-\ln 2$$
$$x(\ln 2+2\ln 5)=\ln 5-\ln 2$$
$$x=\frac{\ln 5-\ln 2}{\ln 2+2\ln 5}$$
$$x\approx 0.234$$

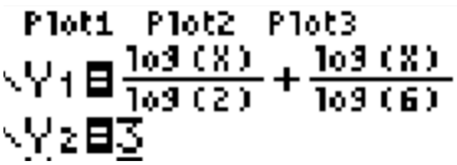
d)

$$\frac{e^x+e^{-x}}{2}=3$$
$$e^x+e^{-x}=6$$
$$e^x(e^x+e^{-x})=6e^x$$
$$e^{2x}+1=6e^x$$
$$e^{2x}-6e^x+1=0$$
$$\left(e^x\right)^2-6e^x+1=0$$
$$e^x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(1)}}{2(1)}$$
$$e^x=\frac{6\pm\sqrt{32}}{2}=\frac{6\pm 4\sqrt{2}}{2}$$
$$e^x=3\pm 2\sqrt{2}$$
$$e^x=3+2\sqrt{2} \quad or \quad e^x=3-2\sqrt{2}$$
$$\ln\left(e^x\right)=\ln\left(3+2\sqrt{2}\right) \quad or \quad \ln\left(e^x\right)=\ln\left(3-2\sqrt{2}\right)$$
$$x=\ln\left(3+2\sqrt{2}\right) \quad or \quad x=\ln\left(3-2\sqrt{2}\right)$$
$$x\approx 1.763 \qquad or \quad x\approx -1.763$$

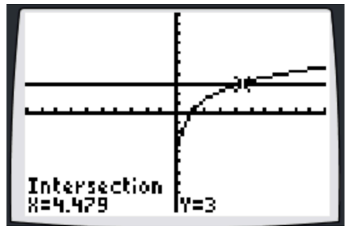
Ex 3: Use the graphing utility of your calculator to solve the following.

a) $\log_2 x + \log_6 x = 3$

Step 1. Place equations in Y=



Step 2: Use intersect feature

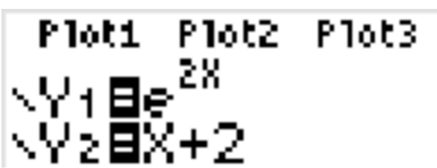


Step 3: Solution

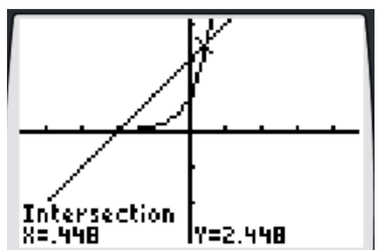
X = 4.479

b) $e^{2x} = x + 2$

Step 1. Place equations in Y=



Step 2: Use intersect feature



Step 3: Solution

X = 0.448