Section 4.7 Date________

Compound Interest

Interest is the cost of using money.

Principal: The total amount of money borrowed	**Rate of Interest**: the amount charged for the use of the principal for a given period of time (written as %, but use decimal for calcs.)
Simple Interest: **I = Prt** P = principal r = interest rate as a decimal (per annum) t = # years the money is borrowed	**Payment Period:** how long before interest is calculated **Annually**: once per year **Semiannually**: twice per year **Quarterly**: four times each year **Monthly**: 12 times each year **Daily**: 365 times each year
Present Value: The amount of principal at the beginning of a loan or investment	**Accumulated (Future) Value**: The amount of money at the end of a loan or investment
Compound Interest: The interest paid on the principal and previously earned interest $A = P\left(1+\frac{r}{n}\right)^{nt}$	**Continuous Compounding**: The money accrued for an infinite number of payment periods $A = Pe^{rt}$
Effective Rate of Interest: the interest rate that is equivalent to the amount of simple interest earned in one year	**Present Value Formulas**: $P = A\left(1+\frac{r}{n}\right)^{-nt}$ $P = Ae^{-rt}$

Zero Coupon Bond:
A bond that is sold now at a discount and will pay its face value at the time when it matures.
No interest payments are made.

Ex 1 Find the amount in each problem.

a) What is the amount of money that you'd have if you invested $50 at an interest rate 6% compounded monthly after a period of 3 years? (#4)

P = 50
R = 0.06
N = 12
T = 3

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 50\left(1+\frac{0.06}{12}\right)^{(12)(3)}$$

$$A = 59.83$$

You would have $59.83 after 3 years.

b) What is the amount of money that you'd have if you invested $100 at an interest rate of 12% compounded continuously after a period of 3¾ years? (#14)

P = 100
R = 0.12
T = 3.75

$$A = Pe^{rt}$$

$$A = (100)e^{(0.12)(3.75)}$$

$$A = 156.83$$

You would have $156.83 after 3.75 years.

Example 2

a) How much principal would you need to invest to get $800 after 3½ years at 7% compounded monthly? (#16)

A = 800
P = ?
R = 0.07
N = 12
T = 3.5

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$800 = P\left(1+\frac{0.07}{12}\right)^{(12)(3.5)}$$

$$P = \frac{800}{\left(1+\frac{0.07}{12}\right)^{(12)(3.5)}}$$

$$P = 626.6095$$

To have $800 after 3.5 years, you would need to invest $626.61.

b) What interest rate compounded quarterly will give an effective interest rate of 7%? (#24)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$107 = 100\left(1 + \frac{r}{4}\right)^{(4)(1)}$$

$$1.07 = \left(1 + \frac{r}{4}\right)^{4}$$

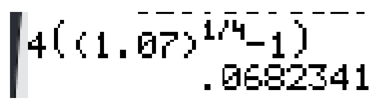
$$\sqrt[4]{1.07} = 1 + \frac{r}{4}$$

$$\sqrt[4]{1.07} - 1 = \frac{r}{4}$$

$$4\left(\sqrt[4]{1.07} - 1\right) = r$$

$$0.0682 \approx r$$

For P, choose a number like 100 or 1000. This way, it's easy to mentally determine 7% of that number, which will give you the amount you would have, A.



An interest rate of 6.82% compounded quarterly would have an effective rate of 7%.

Ex 3 How long does if take for an investment to double in value if it is invested at 10% per annum compounded monthly? Compounded continuously? (#32)

Monthly compounding

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$2P = P\left(1 + \frac{0.10}{12}\right)^{(12)t}$$

$$2 = \left(1 + \frac{0.10}{12}\right)^{12t}$$

$$\ln 2 = \ln\left(1 + \frac{0.10}{12}\right)^{12t}$$

$$\ln 2 = 12t \ln\left(1 + \frac{0.10}{12}\right)$$

$$\frac{\ln 2}{12} = t \ln\left(1 + \frac{0.10}{12}\right)$$

$$\frac{\ln 2}{12 \ln\left(1 + \frac{0.10}{12}\right)} = t$$

$$6.960 \approx t$$

It would take about 7 years for an investment to double.

Continuous compounding

$$A = Pe^{rt}$$
$$2P = Pe^{(0.10)t}$$
$$2 = e^{(0.10)t}$$
$$\ln 2 = 0.10t$$
$$\frac{\ln 2}{0.10} = t$$
$$6.931 \approx t$$

It would take about 7 years for an investment to double.

<u>Ex 4</u> How much should a $10,000 face value zero-coupon bond, maturing in 10 years, be sold for now if its rate of return is to be 8% compounded annually? (#53)

We want the present value of $10,000.

$$P = A\left(1+\frac{r}{n}\right)^{-nt}$$
$$P = 10000\left(1+\frac{.08}{1}\right)^{-1(10)}$$
$$P \approx 4631.934$$

You should sell the zero-coupon bond for $4631.93

<u>You've Got Problems:</u>

- Page 294 #1-59 (eoo)
- Quiz in 2 classes on 4.6 – 4.8