Section 4.8 Date________

Applications of Exponential Functions

Uninhibited Growth: A model that gives the number *N* of cells in the culture after a time *t* has passed

$N(t) = N_0e^{kt}$, $k > 0$

N_0 = Initial population
k = constant rate of growth

Uninhibited Decay: A model that gives the amount *A* of material present at time t.

$A(t) = A_0e^{kt}$, $k < 0$

A_0 = Initial amount of material present
k = constant rate of decay (negative number)

Ex 1 The number N of bacteria present in a culture at time t (in hours) obeys the function $N(t) = 1000e^{0.01t}$.

a) How many bacteria are in the colony at t = 0 hours?	$N(0) = 1000e^{0.01(0)}$ $N(0) = 1000$ At t = 0 hours, there are 1000 bacteria present.
b) What is the growth rate of the bacteria?	The growth rate is 1%.
c) What is the population after 4 hours?	$N(4) = 1000e^{0.01(4)}$ $N(4) = 1040.811$ After 4 hours, the bacteria's population is about 1041.
d) When will the number of bacteria reach 1700?	Find t when N(t) = 1700 $1700 = 1000e^{0.01(t)}$ $1.7 = e^{0.01(t)}$ $\ln 1.7 = 0.01t$ $\frac{\ln 1.7}{0.01} = t$ $t \approx 53.1$ The number of bacteria would reach 1700 after 53.1 hours.

Ex 2 Iodine 131 is a radioactive material that decays according to the function $A(t) = A_0e^{-0.087t}$, where A_o is the initial amount present at time t (in days). Assume that a scientist has a sample of 100 grams of Iodine 131.

a) What is the decay rate of iodine 131?	The rate of decay of Iodine 131 is 8.7%.
b) How much iodine 131 is left after 9 days?	$A(9) = 100e^{-0.087(9)}$ $A(9) = 45.70$ After 9 days, there will be about 45.7 grams of Iodine 131 remaining.
c) When will 70 grams of Iodine 131 be left?	When A = 70 $70 = 100e^{-0.087(t)}$ $0.7 = e^{-0.087(t)}$ $\ln 0.7 = -0.087t$ $\frac{\ln 0.7}{-0.087} = t$ $t \approx 4.10$ After 4.10 days, there will be 70 grams of Iodine 131 left.
d) What is the half-life of Iodine 131?	Since there are 100 grams of Iodine 131 to begin, find t when A = 50. $50 = 100e^{-0.087(t)}$ $0.5 = e^{-0.087(t)}$ $\ln 0.5 = -0.087t$ $\frac{\ln 0.5}{-0.087} = t$ $t \approx 7.97$ The half-life of Iodine 131 is about 7.97 days.

Newton's Law of Cooling: The temperature of a heated object at a given time t can be modeled by the function…

$$\mathbf{u(t) = T + (u_0 - T)e^{kt}},\ k < 0$$

T = the constant temperature of the surrounding medium
u_0 is the initial temperature of the heated object
k is a negative constant

<u>Ex 3</u> A thermometer reads 72°F, and it's put into a refrigerator where the temperature is a constant 38°F. (#14)

a) If the thermometer reads 60°F after 2 minutes, what will it read after 7 minutes?

Step 1: Find k when given T = 38, u_0 = 72, u(t) = 60, t = 2.

$$60 = 38 + (72 - 38)e^{k(2)}$$
$$22 = 34e^{2k}$$
$$\frac{22}{34} = e^{2k}$$
$$\ln\left(\frac{22}{34}\right) = 2k$$
$$\frac{\ln(22/34)}{2} = k$$

Step 2. Plug in k for t = 7.

$$u(7) = 38 + (72 - 38)e^{\left(\frac{\ln(22/34)}{2}\right)(7)}$$
$$u(7) = 38 + 34e^{\left(\frac{\ln(22/34)}{2}\right)(7)}$$
$$u(7) \approx 45.4$$

After 7 minutes, the thermometer reads $45.4°F$.

b) How long will it take before the thermometer reads 39°F? (find T when u = 39)

$$39 = 38 + (72 - 38)e^{\left(\frac{\ln(22/34)}{2}\right)(t)}$$
$$1 = 34e^{\left(\frac{\ln(22/34)}{2}\right)(t)}$$
$$1/34 = e^{\left(\frac{\ln(22/34)}{2}\right)(t)}$$
$$\ln(1/34) = \left(\frac{\ln(22/34)}{2}\right)t$$
$$\frac{2\ln(1/34)}{\ln(22/34)} = t$$
$$t \approx 16.2$$

After 16.2 minutes, the thermometer will read 39 degrees.

c) What time will elapse before the thermometer reads 45°F?

$$45 = 38 + (72 - 38)e^{\left(\frac{\ln(22/34)}{2}\right)(t)}$$

$$7 = 34e^{\left(\frac{\ln(22/34)}{2}\right)(t)}$$

$$7/34 = e^{\left(\frac{\ln(22/34)}{2}\right)(t)}$$

$$\ln(7/34) = \left(\frac{\ln(22/34)}{2}\right)t$$

$$\frac{2\ln(7/34)}{\ln(22/34)} = t$$

$$t \approx 7.3$$

After 7.3 minutes, the thermometer will read 45 degrees.

d) What do you notice about the temperature as time passes?

The temperature gets closer to 38 degrees as time passes.

The **logistical growth model** is an exponential function that models situations where growth is limited.

Carrying capacity (c) – the maximum amount that the situation can support.

Logistical Growth Model: The population P after time t obeys the equation

$$P(t) = \frac{c}{1 + ae^{-bt}},$$

where a, b, and c are constants with b > 0 and c > 0.

Ex 4 Six American bald eagles are captured, transported to Montana and set free. Based on previous experience, the scientists expect the population, P(t), to grow according to the model $P(t)=\frac{500}{1+83.33e^{-0.162t}}$ for t years after the introduction. (#24)

a) What is the carrying capacity of the environment?	$P(t)=\frac{500}{1+83.33e^{-0.162t}}$ $\lim_{t\to\infty}\frac{500}{1+83.33e^{-0.162t}}$ $\lim_{t\to\infty}\frac{500}{1+0}$ $\lim_{t\to\infty}P(t)=500$ The carrying capacity of the environment is 500 bald eagles.
b) What is the predicted population of the eagles in 20 years?	$P(20)=\frac{500}{1+83.33e^{-0.162(20)}}$ $P(20)\approx 117$ In 20 years, there will be 117 eagles.
c) When will the population be 300?	$300=\frac{500}{1+83.33e^{-0.162(t)}}$ $300(1+83.33e^{-0.162(t)})=500$ $1+83.33e^{-0.162(t)}=\frac{500}{300}$ $83.33e^{-0.162(t)}=\frac{5}{3}-1$ $83.33e^{-0.162(t)}=\frac{2}{3}$ $e^{-0.162(t)}=\frac{2/3}{83.33}$ $-0.162t=\ln\left(\frac{2/3}{83.33}\right)$ $t=\frac{\ln\left(\frac{2/3}{83.33}\right)}{-0.162}$ $t\approx 29.8$ The bald eagle population will reach 300 in 29.8 years.

You've Got Problems:

Pg 304: #1-23 (odds)
Quiz next class on 4.6-4.8 (all calculator active)
Ch 4 Test in 3 classes