

$$\sqrt{x+10} = x-2$$

How do you know
you're RIGHT?

$$\sqrt{x+10} = x-2$$

$$(x-2)(x-2)$$

$$x+10 = x^2-4x+4$$

$$0 = x^2-5x-6$$

$$0 = (x-6)(x+1)$$

$$x-6=0$$

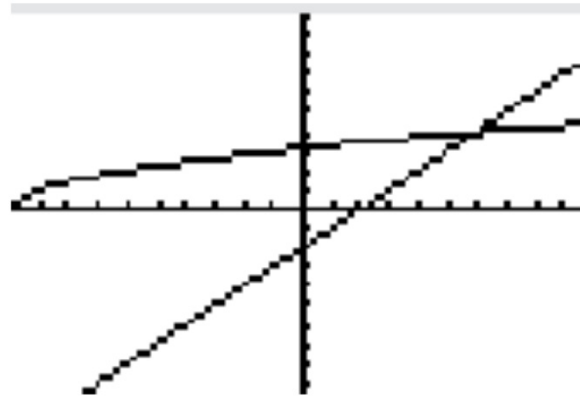
$$x=6$$

$$x+1=0$$

$$x=-1$$

CTS
Factoring
Quad.
Formula

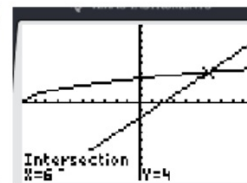
Plot1 Plot2 Plot3
 $Y_1 = \sqrt{X+10}$
 $Y_2 = X-2$



X	Y1	Y2
3	3.4641	0
3.6056	3.6056	1
3.7417	3.7417	1
3.873	3.873	1
4	4	2
4.1231	4.1231	2
4.2426	4.2426	2

CALCULATE
 1: value
 2: zero
 3: minimum
 4: maximum

CALCULATE
 1: value
 2: zero
 3: minimum
 4: maximum
 5: intersect
 6: dy/dx
 7: ∫ f(x) dx



$X=6$

X	Y1	Y2
3	3.4641	0
0	3.1623	-2
1	3.3166	-1

$$\frac{2}{x} = \frac{3}{x-2} - 1$$

$$\frac{2}{x} = \frac{3}{x-2} - \frac{x-2}{x-2}$$

$$\frac{(x-2)}{(x-2)} \frac{2}{x} = \frac{5-x}{x-2} \frac{(x)}{(x)}$$

$$\frac{2x-4}{x(x-2)} = \frac{5x-x^2}{x(x-2)}$$

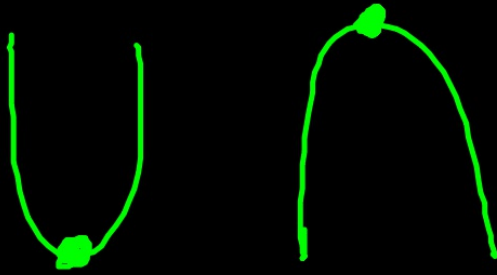
$$2x-4 = 5x-x^2$$

$$x^2-3x-4=0$$

$$(x-4)(x+1)=0$$

$$x-4=0 \quad x+1=0$$

$$x=4 \quad x=-1$$



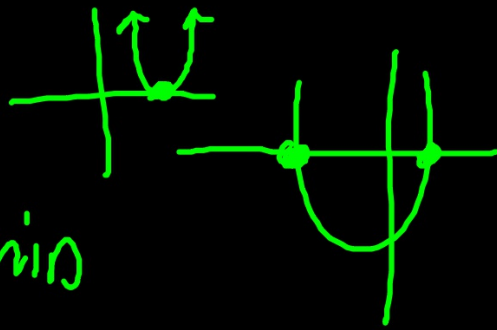
Symmetry

max/min

Vertex

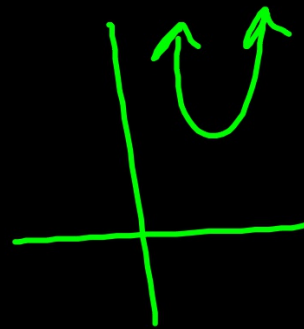
degree = 2

CURVE



Vertex: max/min

Quadratic FORMULA: x-int, roots, zeros, solns



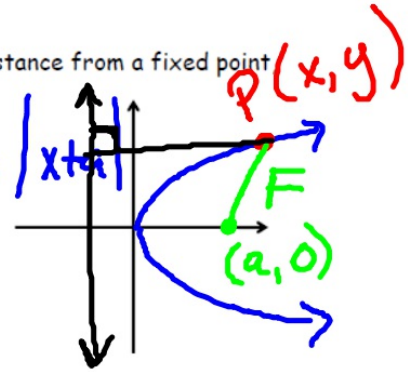
Objective: Students will understand what it means to describe, graph and write the equation of a parabola.

Parabolas

Parabola: collection of all points P in a plane that are the same distance from a fixed point the focus F, and a fixed line (the directrix D)

$$d(F, P) = d(P, D)$$

$$\sqrt{(x-a)^2 + (y-0)^2} = |x+a|$$



$$F(a, 0)$$

$$P(x, y)$$

$$D = (-a, y)$$

$$(x-a)^2 + y^2 = (x+a)^2$$





$$y^2 = (x+a)^2 - (x-a)^2$$

$$y^2 = (x+a)(x+a) - (x-a)(x-a)$$

$$y^2 = x^2 + 2ax + a^2 - (x^2 - 2ax + a^2)$$

$$y^2 = 4ax$$

Vertex at $(0, 0)$, $a > 0$

<p>Opens up $x^2 = 4ay$ </p> <p>Focus: $(0, a)$</p> <p>Directrix: $y = -a$</p>	<p>Opens down $x^2 = -4ay$ </p> <p>Focus: $(0, -a)$</p> <p>Directrix: $y = a$</p>
<p>Opens right $y^2 = 4ax$ </p> <p>Focus: $(a, 0)$</p> <p>Directrix: $x = -a$</p>	<p>Opens left $y^2 = -4ax$ </p> <p>Focus: $(-a, 0)$</p> <p>Directrix: $x = a$</p>

latus rectum - a segment that goes through the focus, and its endpoints are points on the parabola. The endpoints are a distance $\pm 2a$ from the focus.

length of $4a$
 $\perp.R. =$

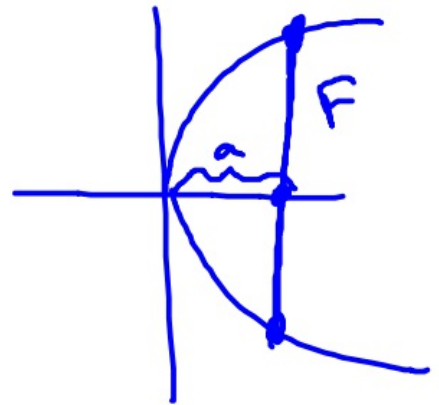
$$y^2 = 4ax$$

$$y^2 = 4a \cdot a$$

$$y^2 = 4a^2$$

$$y = \pm \sqrt{4a^2}$$

$$y = \pm 2a$$



Ex 1 Discuss each equation. (So, find the vertex, focus and directrix.)

a) $y^2 = 8x$

$$4ax = 8x$$

$$a = \frac{8x}{4x}$$

$$a = 2$$

$$V: (0, 0)$$

$$F: (2, 0) \quad (a, 0)$$

$$D: x = -2 \quad x = -a$$

b) $x^2 = -\frac{1}{2}y$

$$-4ay = -\frac{1}{2}y$$

$$-4a = -\frac{1}{2}$$

$$a = \frac{1}{8}$$

$$V: (0, 0)$$

$$F: (0, -\frac{1}{8}) \quad (0, -a)$$

$$D: y = \frac{1}{8} \quad y = a$$

Ex 2 Find the equation of the parabola with vertex at $(0, 0)$; axis of symmetry the x-axis; and contains the point $(2, 3)$.

$$y^2 = 4ax$$

$$(3)^2 = 4a(2)$$

$$9 = 8a$$

$$\frac{9}{8} = a$$

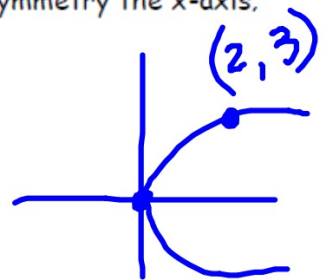
① Find a

$$y^2 = 4ax$$

$$y^2 = 4\left(\frac{9}{8}\right)x$$

$$y^2 = \frac{9}{2}x$$

② Find general equation



Vertex at (h, k) ... Use patterns to describe the focus and directrix.

Opens up

$$(x-h)^2 = 4a(y-k)$$

Opens down

$$(x-h)^2 = -4a(y-k)$$

Opens right

$$(y-k)^2 = 4a(x-h)$$

Opens left

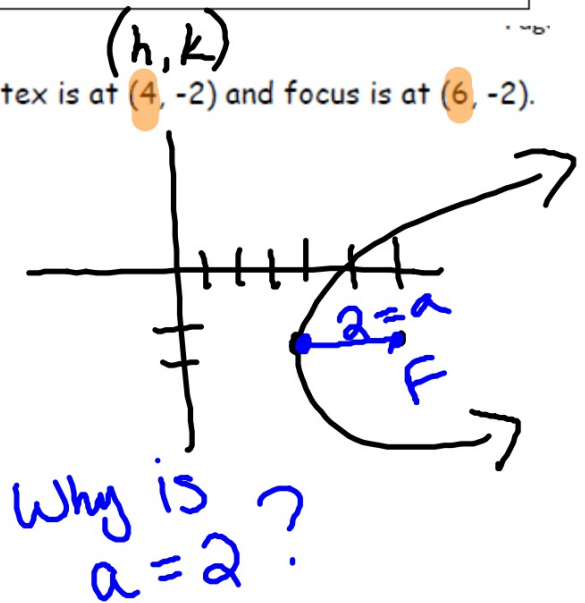
$$(y-k)^2 = -4a(x-h)$$

Ex 3 Find the equation of the parabola whose vertex is at $(4, -2)$ and focus is at $(6, -2)$.

$$(y-k)^2 = 4a(x-h)$$

$$(y+2)^2 = 4(2)(x-4)$$

$$(y+2)^2 = 8(x-4)$$



Ex 4 Discuss the equation:

$$y^2 + 12y = -x + 1$$

V, F, D, LR

$$\left[\frac{1}{2}(12)\right]^2$$

CTS
😊

$$y^2 + 12y + 36 = -x + 1 + 36$$

$$(y+6)^2 = -x + 37$$

want this form \Rightarrow

$$(y-k)^2 = 4a(x-h)$$

$$(y-6)^2 = -1(x-37)$$

$$-1 = 4a$$

$$-\frac{1}{4} = a$$

This gets vs the value of a

V: $(37, 6)$

F: $(37 - \frac{1}{4}, 6) = (36\frac{3}{4}, 6)$

D: $x = 37\frac{1}{4}$

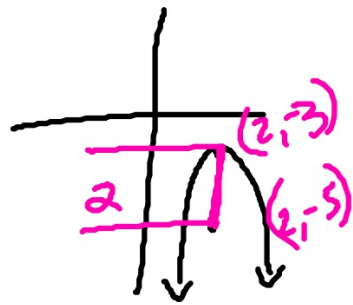
LR: $y = \pm 2a$

$$y = \pm 2\left(-\frac{1}{4}\right)$$

$$y = \pm \frac{1}{2}$$

$$\left(\right)$$

29 $V = \begin{pmatrix} x \\ 2 \\ -3 \end{pmatrix}$
 $F = \begin{pmatrix} a \\ 2 \\ -5 \end{pmatrix}$



$$(x-h)^2 = -4a(y-k)$$

$$(x-2)^2 = -4(2)(y+3)$$

$$(x-2)^2 = -8(y+3)$$

①

$$y^2 = 16x$$

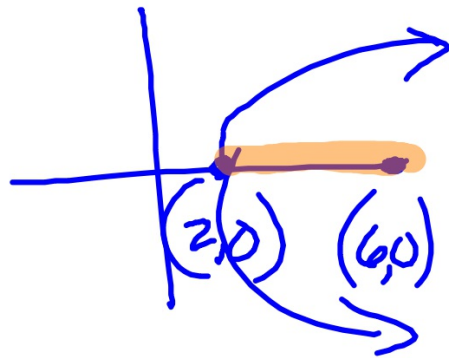
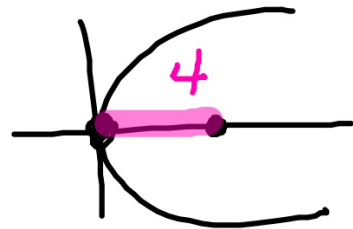
$$V(0,0)$$

$$F(4,0)$$

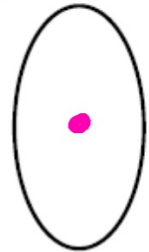
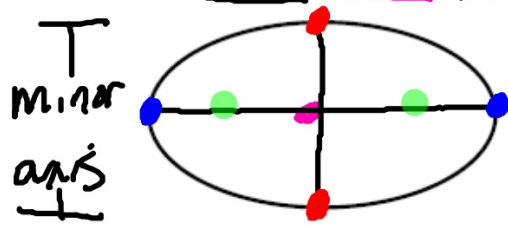
$$y^2 = 4ax$$

$$y^2 = 4(4)x$$

$$y^2 = 16x$$





Ex 1 Label the ellipse's major axis, center, minor axis, vertices and co-vertices.



foci

Equation of an Ellipse: center @ (0, 0)

<p>Foci @ $(\pm c, 0)$ & vertices @ $(\pm a, 0)$</p>  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ <p>where $a > b > 0$ and $b^2 = a^2 - c^2$</p>	<p>Foci @ $(0, \pm c)$ & vertices @ $(0, \pm a)$</p>  $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ <p>where $a > b > 0$ and $b^2 = a^2 - c^2$</p>
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FYI: "Discuss the equation" in ellipses means find the center, major axis, foci, vertices and co-vertices.

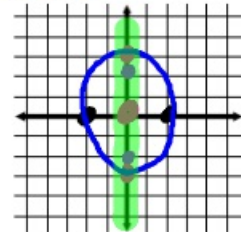
Ex 2 Graph the equation of the conic section. Find the vertices, co-vertices and foci.

$$\frac{y^2}{9} + \frac{x^2}{4} = 1$$

$\frac{4y^2 + 9x^2 = 36}{36 \quad 36 \quad 36}$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$b^2 = 4$ $a^2 = 9$
 $b = \pm 2$ $a = \pm 3$



foci

$$c^2 = a^2 - b^2$$

$$c^2 = 9 - 4$$

$$c^2 = 5$$

$$c = \pm \sqrt{5}$$

$$(0, \sqrt{5}) \quad (0, -\sqrt{5})$$

$$C = (0, 0)$$

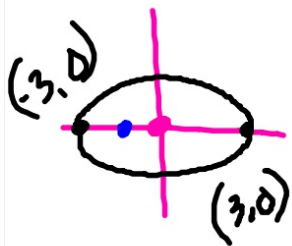
$$V = (0, 3) \quad (0, -3)$$

$$C V = (2, 0) \quad (-2, 0)$$

Ex 3 Find an equation for each ellipse given...

a) Center at $(0, 0)$; focus at $(-1, 0)$; vertex at $(-3, 0)$

b) Foci at $(0, \pm 2)$; major axis measures 8



$$\frac{x^2}{9} + \frac{y^2}{8} = 1$$

$$a = 3$$

$$a^2 = 9$$

$$c^2 = a^2 - b^2$$

$$(-1)^2 = 9 - b^2$$

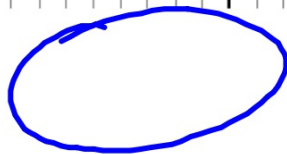
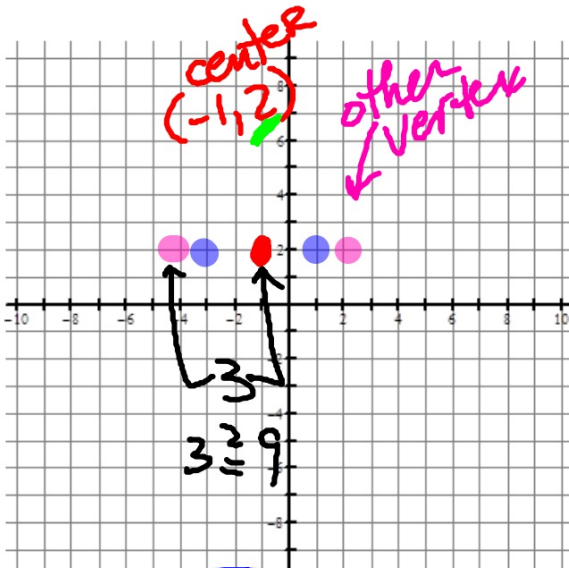
$$1 = 9 - b^2$$

$$8 = b^2$$

Equation of an Ellipse: center @ (h, k)

<p>Foci @ $(h \pm c, k)$ & vertices @ $(h \pm a, k)$</p> $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ <p>where $a > b > 0$ and $b^2 = a^2 - c^2$</p>	<p>Foci @ $(h, k \pm c)$ & vertices @ $(h, k \pm a)$</p> $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ <p>where $a > b > 0$ and $b^2 = a^2 - c^2$</p>
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Ex 4 Find an equation of an ellipse whose foci are (1, 2) and (-3, 2) & whose vertex is (-4, 2)



$$\frac{(x+1)^2}{9} + \frac{(y-2)^2}{5} = 1$$

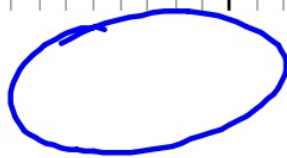
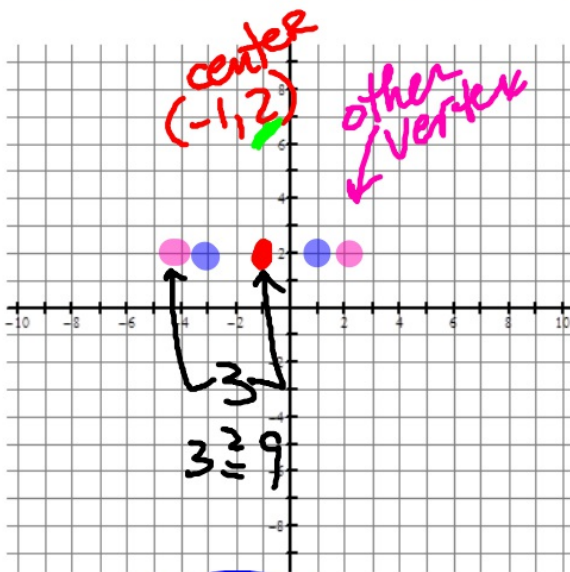
or

$$c^2 = a^2 - b^2 \quad h = -1$$

Equation of an Ellipse: center @ (h, k)

<p>Foci @ $(h \pm c, k)$ & vertices @ $(h \pm a, k)$</p> $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ <p>where $a > b > 0$ and $b^2 = a^2 - c^2$</p>	<p>Foci @ $(h, k \pm c)$ & vertices @ $(h, k \pm a)$</p> $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ <p>where $a > b > 0$ and $b^2 = a^2 - c^2$</p>
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Ex 4 Find an equation of an ellipse whose foci are (1, 2) and (-3, 2) & whose vertex is (-4, 2).



$$\frac{(x+1)^2}{9} + \frac{(y-2)^2}{5} = 1$$

$$c^2 = a^2 - b^2$$

$$(-3, 2)$$

$$h \pm c, k$$

$$h = -1$$

$$-1 \pm c = -3$$

$$-2$$

$$\pm c = -2$$

Ex 5 Discuss each equation. (So, find the center, foci, vertices, co-vertices.)

a) $9(x - 3)^2 + (y + 2)^2 = 18$

$$\frac{(x-3)^2}{2} + \frac{(y+2)^2}{18} = 1$$



center: $(3, -2)$

$a = \sqrt{18} = \sqrt{9} \sqrt{2} = 3\sqrt{2}$

$b = \sqrt{2} =$

b) $4x^2 + 3y^2 + 8x - 6y = 5$

Vertices $(h, k \pm a)$

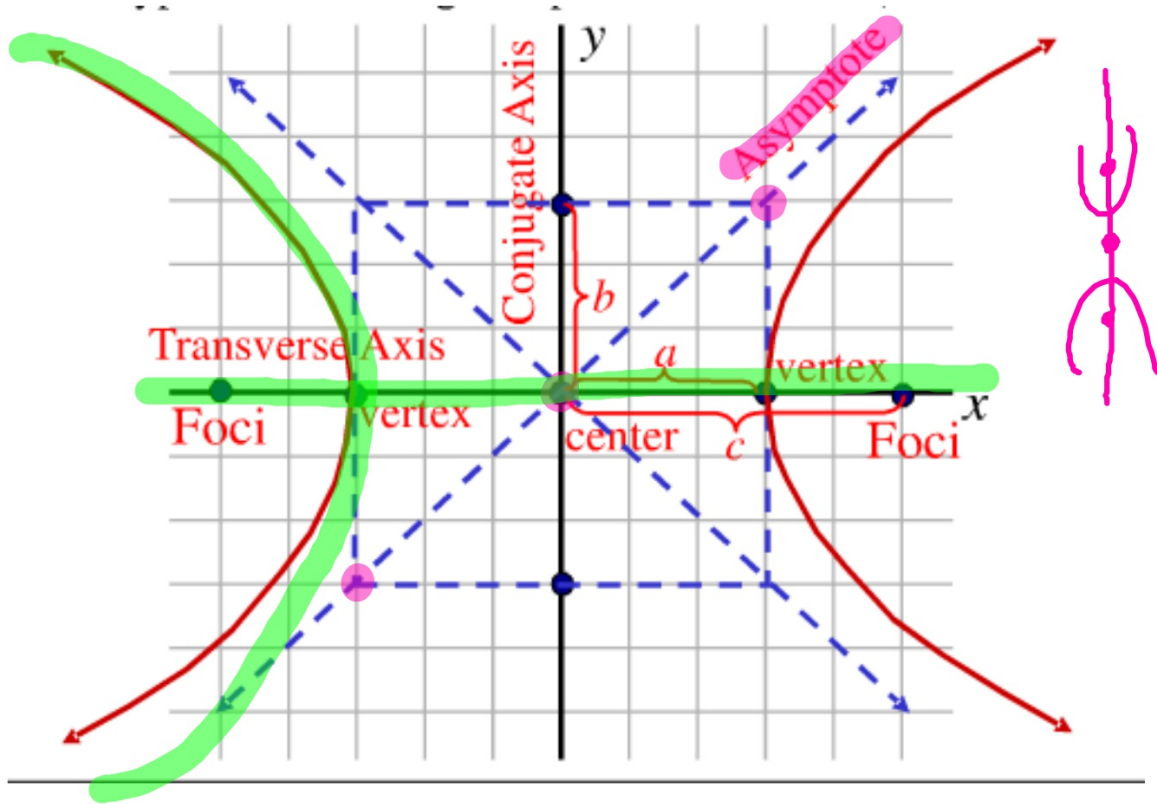
$(\underline{3}, -2 + 3\sqrt{2})$

$(\underline{3}, -2 - 3\sqrt{2})$

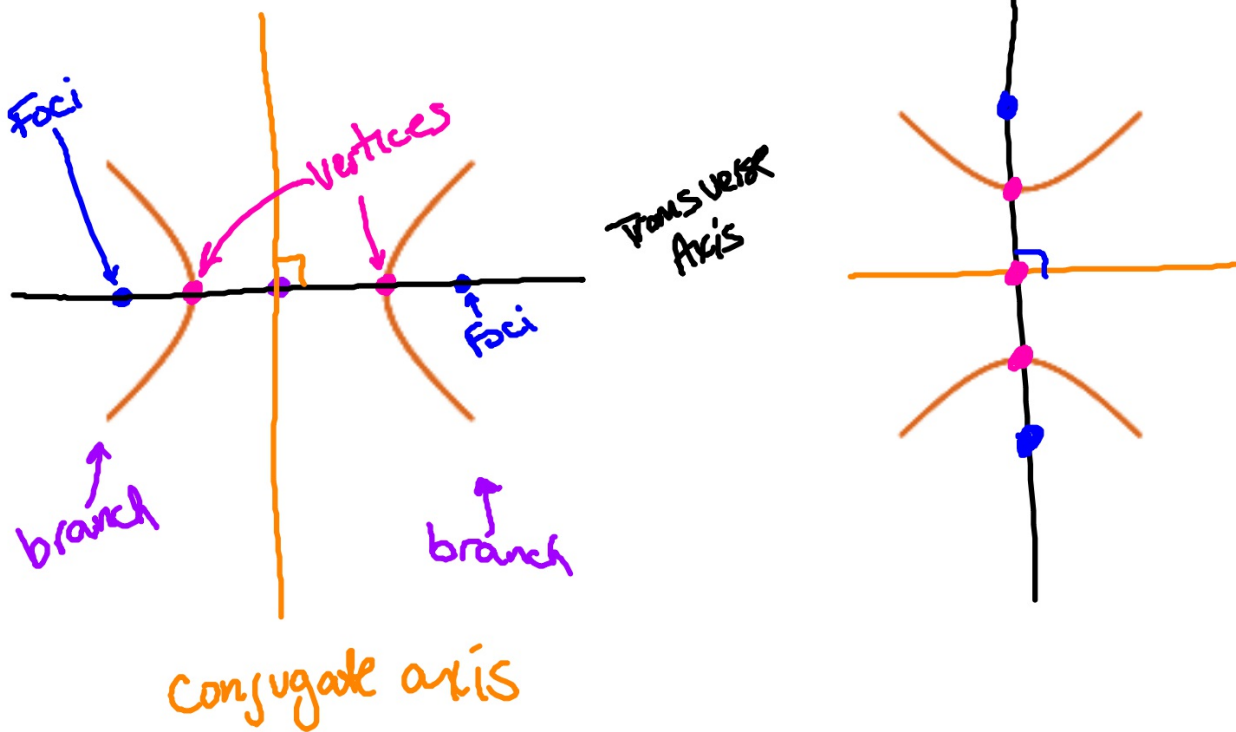
co-vertices use
y values are the same

$(3 - \sqrt{2}, \underline{-2})$

$(3 + \sqrt{2}, \underline{-2})$



Ex 1 Label the axes, center, branches, and vertices of the hyperbola.



Equation of a Hyperbola: center @ (0, 0)

Foci @ $(\pm c, 0)$ & vertices @ $(\pm a, 0)$

Transverse axis parallel to x-axis



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where $b^2 = c^2 - a^2$

$$c^2 = a^2 + b^2$$

Note: Asymptotes are $y = \pm \frac{b}{a}x$.

Foci @ $(0, \pm c)$ & vertices @ $(0, \pm a)$

Transverse axis parallel to y-axis



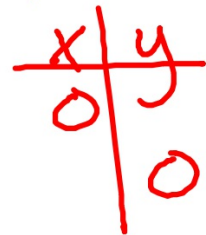
$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

where $b^2 = c^2 - a^2$

$$a^2 + b^2 = c^2$$

Note: Asymptotes are $y = \pm \frac{a}{b}x$.

FYI: "Discuss the equation" for hyperbolas means that you'll find the center, transverse axis, foci, and vertices that hyperbola.



Remember: How would you find the x-intercepts and y-intercepts of a hyperbola?

$(x, 0)$ $(0, y)$

Ex 2 Graph the hyperbola. Then, find its center, foci, vertices and asymptotes.

$$\frac{x^2}{4} - \frac{y^2}{25} = 1$$

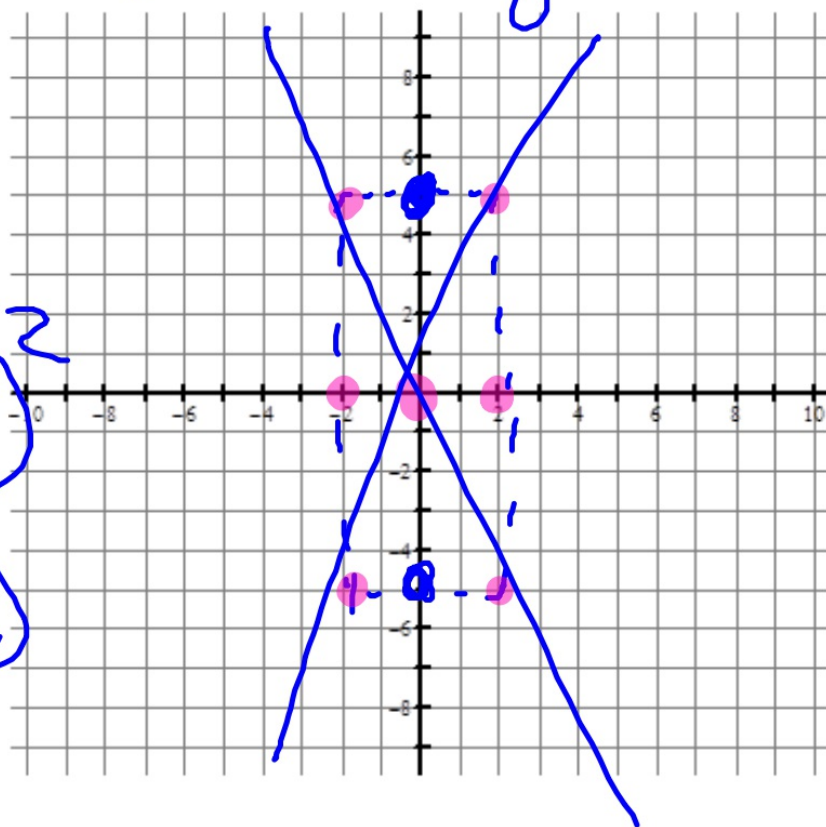
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$\left(\begin{array}{c} \text{>} \\ \text{<} \end{array} \right)$
 $\left(\begin{array}{c} \cup \\ \cap \end{array} \right)$

a b

$$(x+3)^2 - (y-2)^2$$

$$(h, k) (-3, 2)$$



Ex 2 Graph the hyperbola. Then, find its center, foci, vertices and asymptotes.

$$\frac{(x-0)^2}{4} - \frac{(y-0)^2}{25} = 1$$

center: $(0, 0)$

$$a^2 = 4 \Rightarrow a = 2$$

$$b^2 = 25 \Rightarrow b = 5$$

vertices: $(-2, 0)$ $(2, 0)$

Foci $c^2 = a^2 + b^2$

$$c^2 = 4 + 25$$

$$c^2 = 29$$

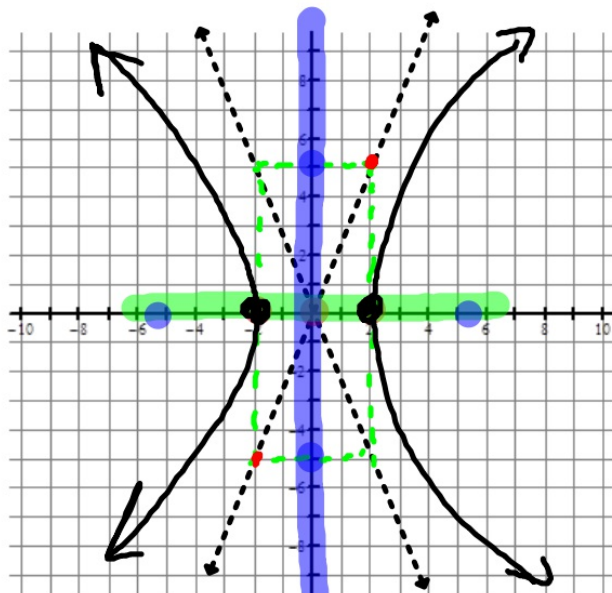
$$c = \pm \sqrt{29}$$

$$\left(\sqrt{29}, 0 \right)$$

$$\left(-\sqrt{29}, 0 \right)$$

Asymptotes

$$y = \pm \frac{5}{2} x$$



be sure asymptotes are dashed ☺

start from

center

Equation of a Hyperbola: center @ (h, k)

Foci @ $(h \pm c, k)$ & vertices @ $(h \pm a, k)$

Transverse axis parallel to x-axis

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\text{where } b^2 = c^2 - a^2$$

Note: Asymptotes are $(y-k) = \pm \frac{b}{a}(x-h)$.

Foci @ $(h, k \pm c)$ & vertices @ $(h, k \pm a)$

Transverse axis parallel to y-axis

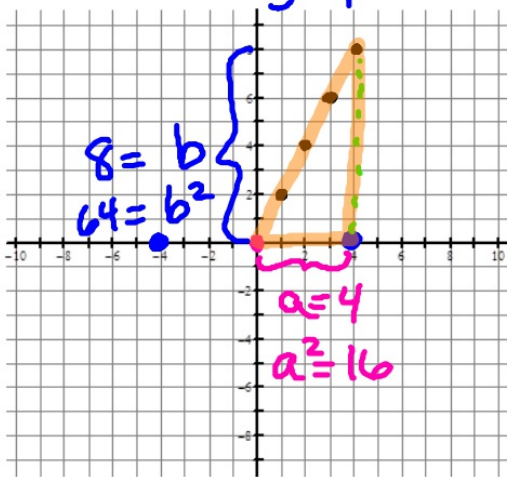
$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\text{where } b^2 = c^2 - a^2$$

Note: Asymptotes are $(y-k) = \pm \frac{a}{b}(x-h)$.



Ex 4 Find an equation for a hyperbola whose vertices are $(4, 0)$ and $(-4, 0)$ and has an asymptote of $y = 2x$. Then state its foci



)C

$$\frac{x^2}{16} - \frac{y^2}{64} = 1$$

Foci

$$c^2 = a^2 + b^2$$

$$c^2 = 16 + 64$$

$$c^2 = 80$$

$$c = \pm \sqrt{80} = \pm 4\sqrt{5}$$

$$c = \pm 4\sqrt{5}$$

$$(4 + 4\sqrt{5}, 0)$$

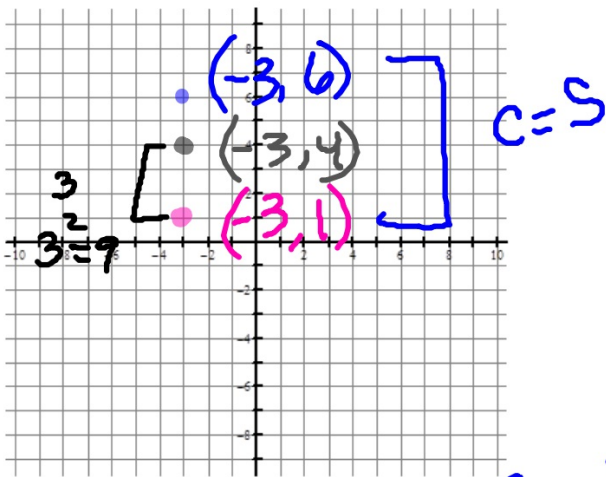
$$(-4 - 4\sqrt{5}, 0)$$

$$\frac{x^2}{16} - \frac{y^2}{64} = 1$$

(

$$x^2 - y^2$$

Ex 5 Find an equation for a hyperbola whose center is at $(-3, 1)$, focus is at $(-3, 6)$ and whose vertex is at $(-3, 4)$.

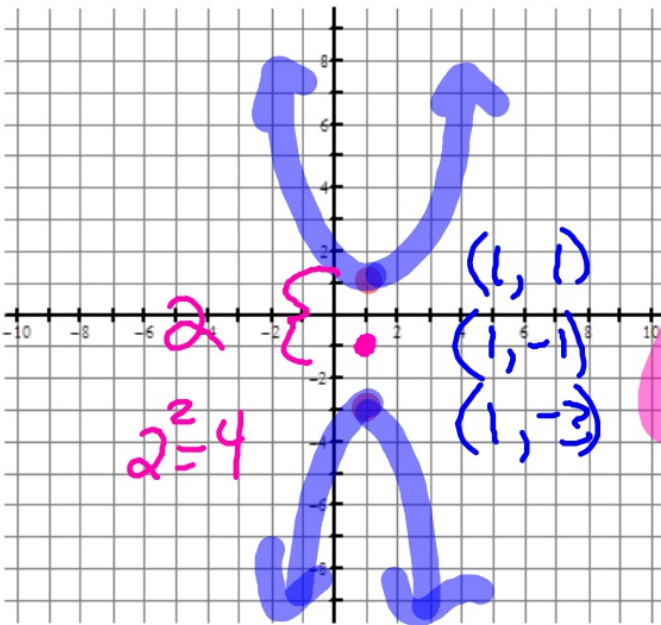


$$\frac{(y-1)^2}{9} - \frac{(x+3)^2}{16} = 1$$

$$c^2 = 25$$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 25 &= 9 + b^2 \\ 16 &= b^2 \\ 4 &= b \end{aligned}$$

Ex 6 Find an equation for a hyperbola whose vertices are at $(1, -3)$ and $(1, 1)$ and whose asymptote is $y+1 = \frac{3}{2}(x-1)$ (h, k)



$(1, -1)$

$$\frac{(y+1)^2}{4} - \frac{(x-1)^2}{16/9} = 1$$

ERROR $\frac{a}{b} \Rightarrow a=3, b=2$

$2^2 = 4$

Substitute 2 for 'a'

$$\frac{a}{b} = \frac{3}{2}$$

$$\frac{2}{b} = \frac{3}{2}$$

$$3b = 4$$

$$b = \frac{4}{3}$$

$$b^2 = \frac{16}{9}$$