Functions Practice Quiz
Calculator Portion

Name ______________________________

1 – 3. Given: $f(x) = \dfrac{6x-2}{2x+1}$ and $g(x) = \dfrac{8x}{3x+5}$

1. Determine the range for $f(x)$. Write your answer using both interval notation and set notation.

$\frac{6x}{2x} = 3$

$\{y \mid y \neq 3\}$

$(-\infty, 3) \cup (3, \infty)$

$y = 3$ is a horizontal asymptote for $f(x)$

2. Determine the domain for $g(x)$. Write your answer using both interval notation and set notation.

$3x + 5 \neq 0$

$3x \neq -5$

$x \neq \frac{-5}{3}$

$\{x \mid x \neq -5/3\}$

$(-\infty, \frac{-5}{3}) \cup (\frac{-5}{3}, \infty)$

$x = \frac{-5}{3}$ is a vertical asymptote for $g(x)$

Functions Practice Quiz
Calculator Portion

Name ______________________________

1 – 3. Given: $f(x)=\dfrac{6x-2}{2x+1}$ and $g(x)=\dfrac{8x}{3x+5}$

inside

3. Find $(g\circ f)(x)$. Determine what is the range of $(g\circ f)(x)$?

$$g(f(x))=\frac{8\left(\frac{6x-2}{2x+1}\right)}{3\left(\frac{6x-2}{2x+1}\right)+5}=\frac{\frac{48x-16}{2x+1}}{\frac{18x-6}{2x+1}+\frac{5(2x+1)}{2x+1}}$$

$$g(f(x))=\frac{\frac{48x-16}{2x+1}}{\frac{18x-6}{2x+1}+\frac{10x+5}{2x+1}}=\frac{\frac{48x-16}{2x+1}}{\frac{28x-1}{2x+1}}$$

$$\boxed{g(f(x))=\frac{48x-16}{28x-1}}$$

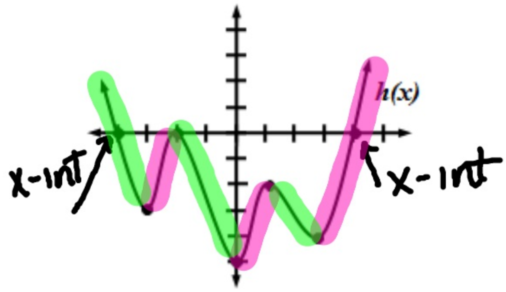
$$\frac{48x}{28x}=\frac{48}{28}=\frac{12}{7}$$

The range of $g(f(x))$ is $\left(-\infty,\frac{12}{7}\right)\cup\left(\frac{12}{7},\infty\right)$

<u>Note:</u> $\frac{12}{7}$ is the horizontal asymptote of $g(f(x))$

Use the graph of $h(x)$ for questions 5 – 8.

5. Give all relative extrema.
6. Give all absolute extrema.
7. Give the intervals the function is increasing on.
8. Give the intervals the function is decreasing on.



⑤ rel min : (-3, -3) (3, -4)
rel max : (-2, 0) (1, -2)

⑥ absolute min : (0, -5)

⑦ $(-3,-2) \cup (0,1) \cup (3,\infty)$

⑧ $(-\infty,-3) \cup (-2,0) \cup (1,3)$

9. If $r(x)=\frac{2}{3}x-5$, determine $r^{-1}(x)$.

$$y=\frac{2}{3}x-5$$

$$x=\frac{2}{3}y-5$$

$$x+5=\frac{2}{3}y$$

$$3x+15=2y$$

$$\frac{3x+15}{2}=y$$

$$\boxed{r^{-1}(x)=\frac{3x+15}{2}}$$

Note: Be sure you use correct notation!

10. Find the average rate of change of $f(x) = 3x^2 - 4$ from x = 1 to x = 8.

$f(8) = 3(8)^2 - 4$

$f(8) = 188$

$f(1) = 3(1)^2 - 4$

$f(1) = -1$

$$\frac{f(8) - f(1)}{8 - 1}$$

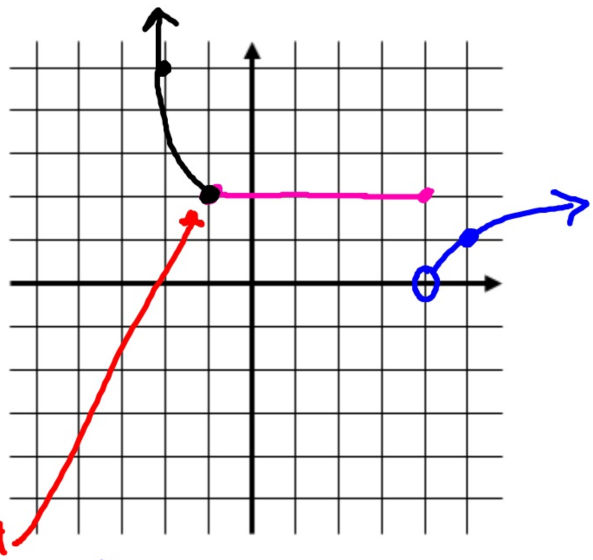
$$\frac{188 - (-1)}{8 - 1}$$

$$\frac{189}{7}$$

$$\boxed{27}$$

11. Graph the following function and give the domain and range in interval notation

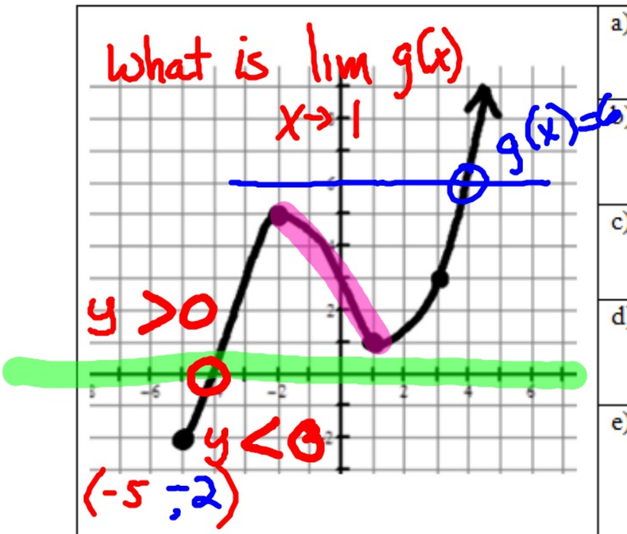
$$g(x) = \begin{cases} x^2 + 1, & x < -1 \\ 2, & -1 \le x \le 4 \\ \sqrt{x-4}, & x > 4 \end{cases}$$



Note: since both pieces meet at the same point, use a solid dot

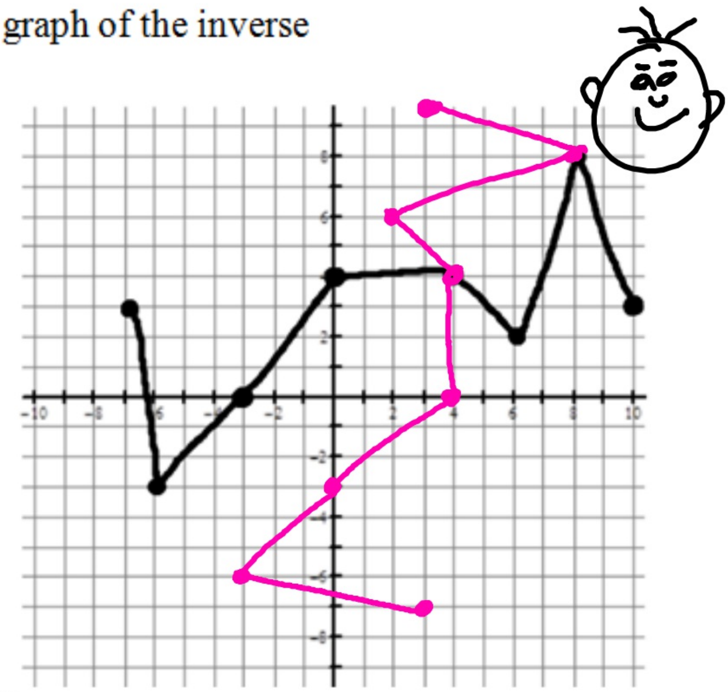
Domain: $(-\infty, \infty)$ Range: $(0, \infty)$

Use the graph of function g given below to answer the following questions. Use interval notation. **Defend your answers.**

<table>
<tr><td rowspan="5">
</td><td>a) What is $g(3)$?
$g(3)=3$</td></tr>
<tr><td>b) Is $g(-2)$ positive or negative?
positive</td></tr>
<tr><td>c) For what intervals is $g(x) < 0$?
$[-5,-4)$</td></tr>
<tr><td>d) What is the domain of g?
$[-5,\infty)$</td></tr>
<tr><td>e) What is the range of g?
$[-2,\infty)$</td></tr>
<tr><td>f) What are the x-intercepts of g?
$(-4,0)$</td><td>g) What is the y-intercept of g?
$(0,3)$</td></tr>
<tr><td>h) How often does the line $y = -1$ intersect the graph of g?
Once or 1</td><td>i) For what values of x does $g(x) = 6$?
$x=4$ $g(4)=6$</td></tr>
<tr><td>j) What are the local maxima?
$(-2,5)$</td><td>k) On which interval(s) is g decreasing?
$(-2,1)$</td></tr>
</table>

2. Given the graph of f, draw the graph of the inverse function, f^{-1}.

$f(x)$	$f^{-1}(x)$
(-7, 3)	(3, -7)
(-6, -3)	(-3, -6)
(-3, 0)	(0, -3)
(0, 4)	(4, 0)
(4, 4)	(4, 4)
(6, 2)	(2, 6)
(8, 8)	(8, 8)
(10, 3)	(3, 10)



Domain: [-7, 10] [-3, 8]

Range: [-3, 8] [-7, 10]

(a) (c) (b)

3. Find the inverse of the function, verify the functions are inverses, then state over what domain and range the functions are inverses. $f(x) = \frac{2x-3}{x+1}$

(a)

$$y = \frac{2x-3}{x+1}$$

$$x = \frac{2y-3}{y+1}$$

$$xy + x = 2y - 3$$

$$xy - 2y = -x - 3$$

$$y(x-2) = -x-3$$

$$y = \frac{-x-3}{x-2}$$

$$\boxed{f^{-1}(x) = \frac{-x-3}{x-2}}$$

Domain $x \neq -1$ for $f(x)$

for inverse, $x \neq 2$

Range

$$f(x) = \frac{2x-3}{x+1}$$

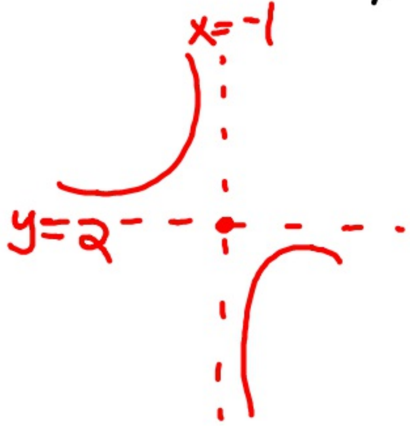
$$\frac{2x}{x} = 2$$

$$y \neq 2$$

$$f^{-1}(x) = \frac{-x-3}{x-2}$$

$$\frac{-x}{x} = -1$$

$$y \neq -1$$



Note: this is the same as $f^{-1}(x) = \frac{x+3}{-x+2}$.

Do you know why?

(b) The domain of f(x) is all real numbers except x = -1.
The range of f(x) is all real numbers except y = 2.

$\mathbb{R}$

The domain of $f^{-1}(x)$ is all real numbers except x = 2.
The range of $f^{-1}(x)$ is all real numbers except y =-1

domain $f(x) = \{x \mid x \neq -1\}$ domain $f^{-1}(x) = \{x \mid x \neq 2\}$

range $f(x) = \{y \mid y \neq 2\}$ range $f^{-1}(x) = \{y \mid y \neq -1\}$

domain $f(x)$

$(-\infty, -1) \cup (-1, \infty)$

range $f(x)$

$(-\infty, 2) \cup (2, \infty)$

domain $f^{-1}(x)$

$(-\infty, 2) \cup (2, \infty)$

range $f^{-1}(x)$

$(-\infty, -1) \cup (-1, \infty)$

C To verify 2 functions are inverses of each other:

1. Show $f(f^{-1}(x)) = x$
2. Show $f^{-1}(f(x)) = x$
3. If you have sucessfully done 1 and 2 Write this conclusion:

Since $f(f^{-1}(x))=f^{-1}(f(x))=x$,
$f(x)$ and $f^{-1}(x)$ are inverse functions.

$$f(x)=\frac{2x-3}{x+1} \qquad f^{-1}(x)=\frac{-x-3}{x-2}$$

$$f(f^{-1}(x))=\frac{2\left(\frac{-x-3}{x-2}\right)-3}{\left(\frac{-x-3}{x-2}\right)+1}=\frac{\frac{-2x-6}{x-2}-\frac{3(x-2)}{x-2}}{\frac{-x-3}{x-2}+\frac{x-2}{x-2}}$$

$$f(f^{-1}(x))=\frac{\frac{-2x-6}{x-2}-\frac{3x-6}{x-2}}{\frac{-x-3}{x-2}+\frac{x-2}{x-2}}=\frac{\frac{-5x}{x-2}}{\frac{-5}{x-2}}=\frac{-5x}{-5}=x$$

$$f^{-1}(f(x))=\frac{-\left(\frac{2x-3}{x+1}\right)-3}{\left(\frac{2x-3}{x+1}\right)-2}=\frac{\frac{-2x+3}{x+1}-\frac{3(x+1)}{x+1}}{\frac{2x-3}{x+1}-\frac{2(x+1)}{x+1}}$$

$$f^{-1}(f(x))=\frac{\frac{-2x+3}{x+1}-\frac{3x+3}{x+1}}{\frac{2x-3}{x+1}-\frac{2x+2}{x+1}}=\frac{\frac{-5x}{x+1}}{\frac{-5}{x+1}}=\frac{-5x}{-5}=x$$

Since $f(f^{-1}(x))=f^{-1}(f(x))=x$,
$f(x)$ and $f^{-1}(x)$ are inverse functions.