Quiz Review 11.1 - 11.3 Key

① $a_n = (-1)^n(2n+9)$

$a_1 = -11$

$a_2 = 13$

$a_3 = -15$

$a_4 = 17$

$a_5 = -19$

② $a_n = n!$

$a_1 = 1$

$a_2 = 2$

$a_3 = 6$

$a_4 = 24$

$a_5 = 120$

③ $a_n = -3n+5$

$a_1 = 2$

$a_2 = -1$

$a_3 = -4$

$a_4 = -7$

$a_5 = -10$

④ $a_n = \frac{n-2}{n^2+1}$

$a_1 = -1/2$

$a_2 = 0$

$a_3 = 1/10$

$a_4 = 2/17$

$a_5 = 3/26$

⑤ $a_1 = 15$, $a_n = 2a_{n-1} - 5$

$a_1 = 15$

$a_2 = 2(15) - 5 = 25$

$a_3 = 2(25) - 5 = 45$

$a_4 = 2(45) - 5 = 85$

$a_5 = 2(85) - 5 = 165$

⑥ $a_1 = -5$ $a_n = a_{n-1} - 6$

$a_1 = -5$

$a_2 = (-5) - 6 = -11$

$a_3 = (-11) - 6 = -17$

$a_4 = (-17) - 6 = -23$

$a_5 = (-23) - 6 = -29$

⑦ Find the explicit formula.

1, 4, 7, 10, ...

$a_1 = 1$ $d = 3$

$a_n = 1 + (n-1)(3)$

$a_n = 1 + 3n - 3$

$a_n = 3n - 2$

⑧ Find the recursive formula.

32, 16, 8, 4, ...

$a_1 = 32$ $r = \frac{1}{2}$

$a_{K+1} = \frac{1}{2} a_K = \frac{a_K}{2}$ (OR) $a_K = \frac{1}{2} a_{K-1} = \frac{a_{K-1}}{2}$

⑨ 3, 6, 9, 12, 15, ... ARITHMETIC sequence, $d = 3$

⑩ -2, 4, -8, 16, -32, ... GEOMETRIC SEQUENCE, $r = -2$

⑪ Find 4 arithmetic means between 5 and 55.

$5, \_\_, \_\_, \_\_, \_\_, 55$ (6 terms)

(a_1 = 5, a_n = 55)

$a_n = a_1 + (n-1)d$

$55 = 5 + (6-1)d$

$55 = 5 + 5d$

$50 = 5d$

$10 = d$ ⟶ 5, 15, 25, 35, 45, 55

(12) Find a formula for a_n for the arithmetic sequence if $a_4 = 22$, $a_{10} = 46$

Alternate Technique

a_1 ↓ , , , 22, , , , , , a_n ↓ 46

$n = 7$ terms

$46 = 22 + (7-1)d$

$46 = 22 + 6d$

$24 = 6d$

$4 = d$

$a_1 = 10 \rightarrow a_n = a_1 + (n-1)d$

$a_n = 10 + (n-1)4$

$\boxed{a_n = 4n + 6}$

(13) Find the n^{th} partial sum of the arithmetic sequence: $8, 20, 32, 44, \ldots, n = 10$

$n = 10$

$a_1 = 8$

$d = 12$

$a_{10} = ?$

$a_n = a_1 + (n-1)d$

$a_{10} = 8 + (10-1)(12)$

$a_{10} = 116$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{10} = \frac{10}{2}(8 + 116)$

$S_{10} = 620$

(14) Find the sum

a) $\sum_{n=25}^{75} 3n$

$a_1 = 3(25) = 75$

$n = 75 - 25 + 1 = 51$

$a_{51} = 3(75) = 225$

$S_{25-75} = \frac{51}{2}(75 + 225)$

$S_{25-75} = 7650$

b) $\sum_{n=250}^{500} (n+3)$

$a_1 = 253$

$n = 251$

$a_{251} = 503$

$S_{250-500} = \frac{251}{2}(253 + 503)$

$S_{250-500} = 94,878$

c) $\sum_{K=1}^{\infty} 2\left(\frac{3}{4}\right)^{K+1}$

$a_1 = 2\left(\frac{3}{4}\right)^2 = \frac{18}{16}$

$r = \left|\frac{3}{4}\right| < 1$

$\sum_{K=1}^{\infty} 2\left(\frac{3}{4}\right)^{K+1} = \frac{a_1}{1-r}$

$= \frac{\frac{18}{16}}{1 - \frac{3}{4}}$

$\sum_{K=1}^{\infty} 2\left(\frac{3}{4}\right)^{K+1} = 9/2$

(15) Find 5 geometric means between 3 and 192

$3, \_, \_, \_, \_, \_, 192$ (7 terms)

$a_n = a_{n-1} r^{n-1}$

$192 = 3r^{7-1}$

$192 = 3r^6$

$64 = r^6$

$\sqrt[6]{64} = r$

$2 = r$

3, 6, 12, 24, 48, 96, 192

(16) Find the mean proportion between -5 and -15.

$\frac{-5}{x} = \frac{x}{-15}$

$x^2 = 75$

$x = \pm\sqrt{75} = \pm 5\sqrt{3}$

We want $-5\sqrt{3}$ since -5 and -15 are negative

(17) Find the partial Sum. Show all work.

a) $5 + 1 + 0.2 + 0.04 + \ldots$ Find S_{12}

$a_1 = 5$ $n = 12$

Sequence is Geometric,

$r = 1/5$

$S_n = \frac{a_1(1 - r^n)}{1 - r}$

$S_{12} = \frac{5(1 - (1/5)^{12})}{1 - (1/5)}$

$S_{12} \approx 6.25$

b) $2 + \frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \ldots$ Find S_{15}

$a_1 = 2$ $n = 15$

$r = (4/3) \div 2 = 2/3$

$S_n = \frac{a_1(1 - r^n)}{1 - r}$

$S_{15} = \frac{2(1 - (2/3)^{15})}{1 - (2/3)}$

$S_{15} \approx 5.9683$

(17) c) $\sum_{n=1}^{5} \frac{n!}{2} = \frac{1}{2}\sum_{n=1}^{5} n! = \frac{1}{2}[1! + 2! + 3! + 4! + 5!]$

$= \frac{1}{2}[1 + 2 + 6 + 24 + 120]$

$= 76.5$

d) $4 + 2 + 0 + (-2) + \cdots$ Find S_{18}

ARITHMETIC series!!

$a_1 = 4$ $\quad a_{18} = 4 + (18-1)(-2)$ $\quad S_n = \frac{n}{2}(a_1 + a_n)$

$d = -2$ $\quad = -30$ $\quad S_{18} = \frac{18}{2}(4 + (-30))$

$n = 18$ $\quad S_{18} = -234$

(18) $\frac{3(4!)}{7!} = \frac{3(4!)}{7 \cdot 6 \cdot 5 \cdot 4!} = \frac{1}{70}$ $\quad (7! = \frac{7 \cdot 6}{2} \cdot 5 \cdot 4!)$

(19) $\frac{(n+1)!}{(n-1)!} = \frac{(n+1)(n)(n-1)!}{(n-1)!} = (n+1)(n) = n^2 + n$

(20) $0.\overline{36} = 0.36\,36\,\overline{36}$

$a_1 = 0.36$

$r = \frac{0.0036}{0.36} = 0.01$

$0.\overline{36} = \frac{0.36}{1 - 0.01} = \frac{0.36}{0.99}$

$0.\overline{36} = \frac{36}{99} = \frac{4}{11}$

$0.\overline{36} = 4/11$

(21) $0.3\overline{18} = 0.31818\overline{18}$

$= 0.3 + 0.018\,18\overline{18}$

$a_1 = 0.018$

$r = \frac{0.00018}{0.018} = 0.01$

$0.3\overline{18} = 0.3 + \frac{0.018}{1 - (.01)}$

$0.3\overline{18} = 0.3 + \frac{18}{990} = \frac{3}{10} + \frac{2}{110}$

$0.3\overline{18} = \frac{33}{110} + \frac{2}{110} = \frac{35}{110}$

$0.3\overline{18} = 7/22$