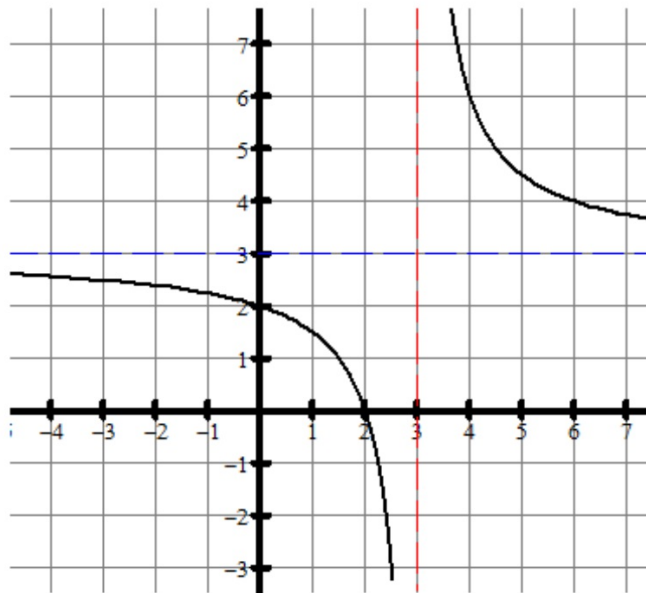


RATIONAL Functions

Analyzing Graphs of Rational Functions

EX 1]



$$\frac{3x-6}{3(x-2)}$$

Vertical Asymptote:

$$x=3$$

Horizontal/Oblique Asymptote:

$$y=3$$

x-intercept:

$$(2,0)$$

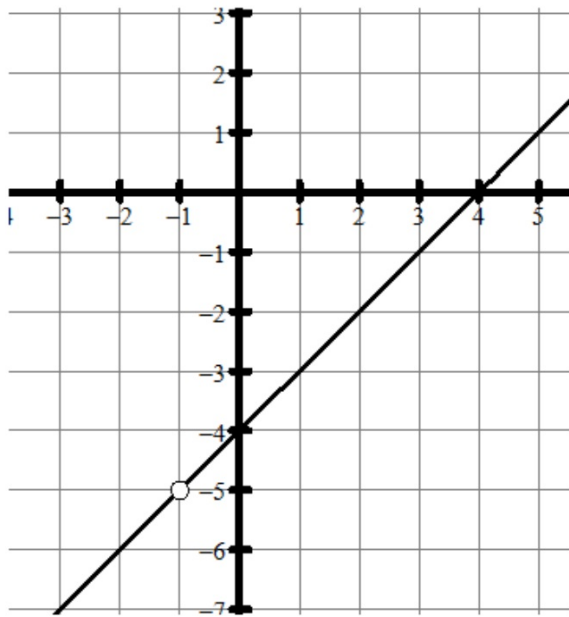
Rational Function's Equation:

$$\frac{3(x-2)}{(x-3)}$$

$$y = \frac{\begin{array}{c} \mu A \\ \downarrow \\ \# \\ \# \end{array} (x - \text{int})}{(V.A)}$$

$$y = \frac{3}{(x-2)(x-3)}$$

EX 2]



Coordinates of Hole:

$$(-1, -5)$$

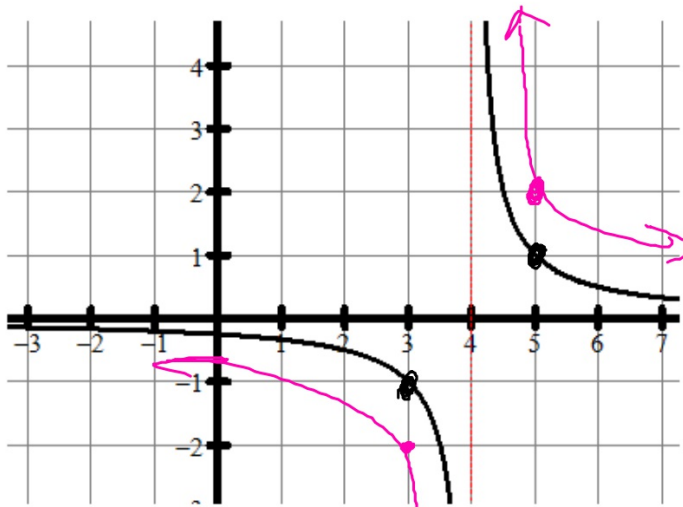
x-intercept: $(4, 0)$

y-intercept: $(0, -4)$

Rational Function's Equation:

$$y = \frac{(x+1)(x-4)}{(x+1)}$$

EX 3]



$$\frac{2}{x-4}$$

$f(x)$

Vertical Asymptote:
 $x = 4$

Horizontal/Oblique Asymptote:
 $y = 0$

Rational Function's Equation:
$$= \frac{1}{(x-4)}$$

$$\frac{x^3 - 6x^2}{x^2 + x}$$

To find
oblique/slant
Asymptote

$$\begin{array}{r} x-7 \\ \hline x^2+x \mid x^3 - 6x^2 - 0x - 0 \\ - (x^3 + x^2) \\ \hline -7x^2 - 0x \end{array}$$

MA

$$\frac{x^3}{x^3} =$$

$$MA = 1$$

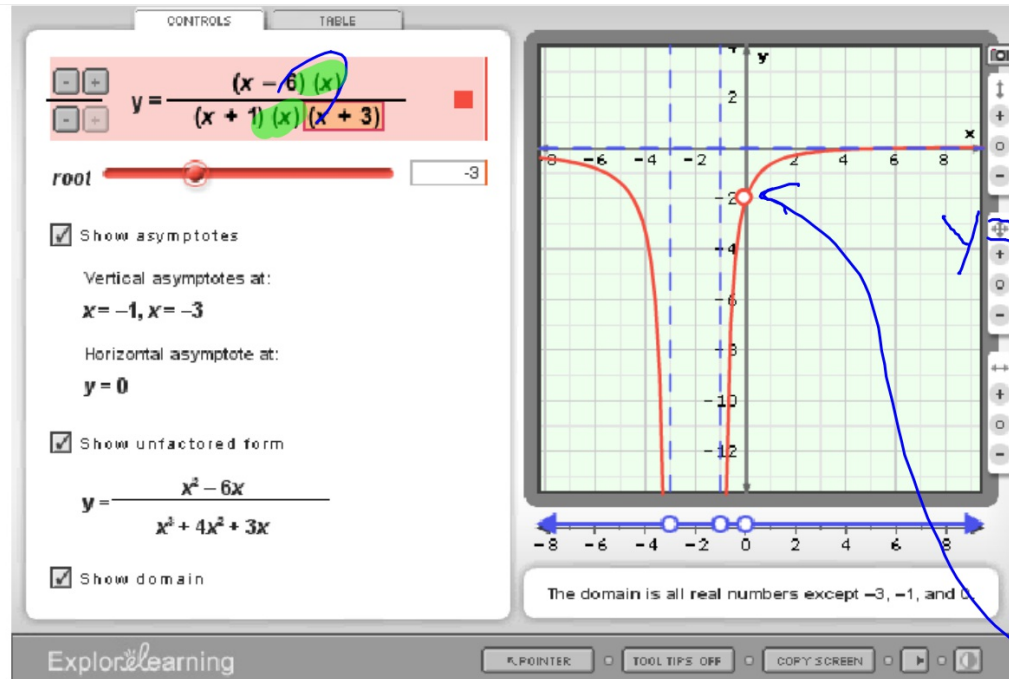
$$\frac{3x^2}{2x^2}$$

$$MA = \frac{3}{2}$$

$$\frac{7x}{5x}$$

$$MA = \frac{7}{5}$$

$$\frac{7x^2}{x^3} \Rightarrow \text{HA: } y=0$$



Hxle

$$\frac{x-6}{(x+1)(x+3)}$$

$$y = \frac{-6}{(1)(3)}$$

$$y = -2$$

$$(0, -2)$$

Graph each of the functions. Include information that identifies each of the following:

- Hole in the graph
- Vertical Asymptote
- Horizontal Asymptote
- Slant Asymptote (find the equation)
- x-intercept
- y-intercept
- key points (use your calc for help)

Show all work on how you obtain each. Be sure you factor correctly!

1. $f(x) = \frac{1}{x+1}$

7. $f(x) = \frac{x^2 - 49}{x^2 - 8x + 7}$

2. $f(x) = \frac{5x+10}{x^2-x-6}$

8. $f(x) = \frac{x-4}{x^2-x-6}$

3. $f(x) = \frac{6x+6}{x^2-3x-4}$

9. $f(x) = \frac{x^2+5x+8}{x+3}$

4. $f(x) = \frac{4x+3}{x+1}$

10. $f(x) = \frac{-x^2+3x+10}{x}$

5. $f(x) = \frac{4}{x^2-6x+5}$

11. $f(x) = \frac{-x^2-2x+24}{2x+8}$

6. $f(x) = \frac{3x}{x^2+x-20}$

12. The cost of producing x units of a product is $C = 0.2x^2 + 10x + 5$, and therefore the average cost per unit is given by

$$C = \frac{C}{x} = \frac{0.2x^2 + 10x + 5}{x}, \quad 0 < x.$$

Sketch the graph of the average cost function, and estimate the number of units that should be produced to minimize the average cost per unit.

Ans. Key for MW

Problem	Hole in graph	VA	HA	SA	x-int	y-int
1	none	$x = 1$	$y = 0$	none	none	$(0, 1)$
2	$(-2, -1)$	$x = 3$	$y = 0$	none	none	$(0, 5/3)$
3	$(-1, -6/5)$	$x = 4$	$y = 0$	none	none	$(0, -3/2)$
4	none	$x = 1$	$y = 4$	none	$(-3/4, 0)$	$(0, 3)$
5	none	$x = 1$ $x = 5$	$y = 0$	none	none	$(0, 4/5)$
6	none	$x = -4$ $x = 5$	$y = 0$	none	$(0, 0)$	$(0, 0)$
7	$(7, 7/3)$	$x = 1$	$y = 1$	none	$(-7, 0)$	$(0, -7)$
8	none	$x = -2$ $x = 3$	$y = 0$	none	$(4, 0)$	$(0, 2/3)$
9	none	$x = 3$	none	$y = x+2$	none	$(0, 8/3)$
10	none	$x = 0$	none	$y = -x+3$	$(5, 0)$ $(-2, 0)$	none
11	none	$x = -4$	none	$y = -1/2x + 1$	$(-6, 0)$ $(4, 0)$	$(0, 3)$