Pre-Calculus Exam Review

Semester 2 Exam

Analytic Trigonometry (Chapter 6)

Find values to answer the following questions.

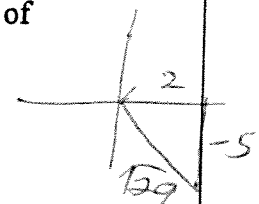
1. If $\tan\alpha = -\frac{3}{4}$ when $\frac{\pi}{2} < \alpha < \pi$ and $\cos\beta = \frac{5}{13}$ when $\frac{3\pi}{2} < \beta < 2\pi$, what is the ***exact*** value of $\tan(\alpha+\beta)$? $= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$$\frac{\left(-\frac{3}{4}\right)+\left(-\frac{12}{5}\right)}{1-\left(-\frac{3}{4}\right)\left(\frac{-12}{5}\right)} = \frac{\frac{-15-48}{20}}{\frac{20-36}{20}} =$$

$$1-\frac{36}{20}$$

$$= \frac{-15-48}{20-36} = \frac{-63}{-16} = \frac{63}{16}$$

2. When P(2, -5) is on the terminal side of θ in standard position, what is the value of $\cos 2\theta$?



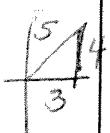
$$\cos 2\theta = 2\cos^2\theta - 1$$

$$= 2\left(\frac{2}{\sqrt{29}}\right)^2 - 1$$

$$= \left(2\cdot\frac{4}{29}\right) - 1$$

$$\frac{8}{29} - \frac{29}{29} = \boxed{\frac{-21}{29}}$$

3. If $\cos\alpha = \frac{3}{5}$ and $\frac{3\pi}{2} < \alpha < 2\pi$, what is the value of $\sin\frac{\alpha}{2}$? $= \pm\sqrt{\frac{1-\cos\alpha}{2}}$

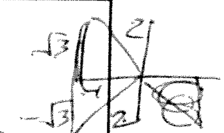


$$= +\sqrt{\frac{1-\frac{3}{5}}{2}} =$$

$$= \sqrt{\frac{\frac{2}{5}}{2}} = \sqrt{\frac{2}{5}\cdot\frac{1}{2}} = \sqrt{\frac{1}{5}}$$

$$= \frac{\sqrt{5}}{5}$$

4. What are the solution(s) of $2\cos^2\theta + 1 = -3\cos\theta$ if θ is in the interval $[0, 2\pi)$?



$\theta' = \frac{\pi}{3}$

$$2\cos^2\theta + 3\cos\theta + 1 = 0$$

$$(2\cos\theta + 1)(\cos\theta + 1) = 0$$

$$\cos\theta = -\frac{1}{2} \qquad \cos\theta = -1$$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \qquad \theta = \pi$$

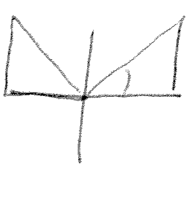
$$\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$$

5. What is the solution of $\sin\theta = 0.8704$ to the nearest hundredth of a degree?

$\theta' = 60.51$

$\sin^{-1}(0.8704)$

$\theta_1 = 60.51°$



$\theta_2 = 180 - 60.51°$

$= 119.49°$

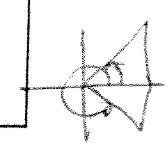
6. What is the solution of $\sec\theta = 1.8492$ to the nearest hundredth of a radian?

$$\cos\theta = \frac{1}{1.8492} = .9994$$

≈ 1 rad

$\boxed{\theta = 1.00}$

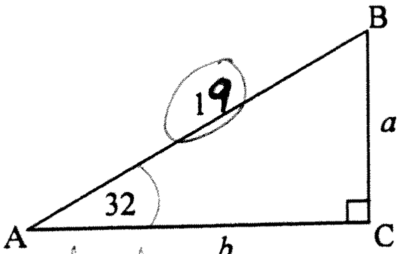
$\theta = 2\pi - 1.00 = \boxed{5.28}$



Applications of Trigonometry (Chapter 7)

Solve each triangle.

7.



$\cos 32 = \frac{b}{(19)}$

$19\cos 32 = b$

$\boxed{16.11 = b}$

$\sin 32 = \frac{a}{19}$

$19\sin 32 = a$

$\boxed{10.07 = a}$

$\angle B = 90 - 32 = 58$

$\alpha = 32^\circ$ $B = 58^\circ$ $C = 90^\circ$

$a = 10.07$ $b = 16.11$ $c = 19$

8. $\alpha = 115°, \beta = 25°, a = 180$

$180 - 115 - 25 = 40^\circ$

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$b = \frac{a \sin B}{\sin A}$

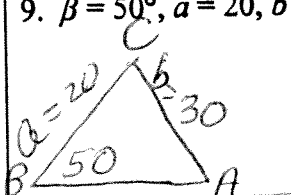
$b = \frac{180 \sin 25^\circ}{\sin 115^\circ} = 83.94$

$c = \frac{180 \sin 40^\circ}{\sin 115^\circ} = 127.66$

$\angle A = 115$ $\angle B = 25$ $\angle C = 40^\circ$

$a = 180$ $b = 83.94$ $c = 127.66$

9. $\beta = 50°, a = 20, b = 30$



$1\ \Delta's$

$\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin 50}{30} = \frac{\sin A}{20}$

$30 \sin A = 20 \sin 50$

$\sin A = \frac{20 \sin(50)}{30}$

$\angle A = \sin^{-1}\left(\frac{20 \sin 50}{30}\right) = \boxed{30.7}$

$C = 180 - 30.7 - 50 = 99.3$

$c = \frac{b \sin \angle C}{\sin B}$

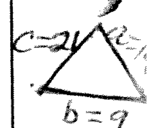
$c = \frac{30 \sin 99.3}{\sin 50}$

$C = 38.65$

$\boxed{\angle A = \alpha = 30.7}$ $\angle B = 50^\circ$ $\angle C = 99.3$

$a = 20$ $b = 30^\circ$ $c = 38.65$

10. $a = 14, b = 9, c = 21$



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 - a^2 - b^2 = -2ab\cos C$

$a^2 + b^2 - c^2 = 2ab\cos C$

$\frac{a^2 + b^2 - c^2}{2ab} = \cos C$

$\angle C = \cos^{-1}\left(\frac{14^2 + 7^2 - 21^2}{2 \cdot 14 \cdot 9}\right)$

$\boxed{\angle C = 130.6^\circ}$

$\frac{\sin 130.6}{21} = \frac{\sin A}{14}$

$\sin A = \frac{14 \sin 130.6}{21} = \angle A = 30.4$

$\boxed{\angle B = 180 - 130.6 - 30.4 = 19}$

$\boxed{\angle A = 30.4}$ $\boxed{\angle B = 19^\circ}$ $\boxed{\angle C = 130.6}$

angles given

$\boxed{Z = \text{integers}}$

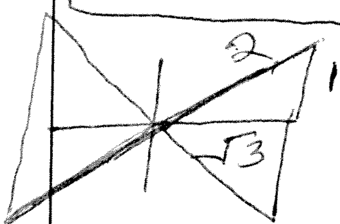
11. Give the general solution for the equation.

$$3\tan^2 x - 1 = 0$$

$\tan^2 x = \frac{1}{3}$

$\boxed{\tan x = \pm\frac{1}{\sqrt{3}}}$ $\boxed{\text{or } \pm\frac{\sqrt{3}}{3}}$

$$\boxed{\begin{aligned} x &= \frac{\pi}{6} + n\pi,\ n \in Z \\ x &= \frac{5\pi}{6} + n\pi,\ n \in Z \end{aligned}}$$



12. Solve the equation for $\theta \in [0, 2\pi)$.

$$2\cos x + 2\sin 2x = 0$$

$2\cos x + 2(2\sin x\cos x) = 0$

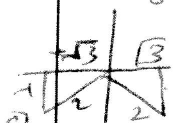
$2\cos x(1 + 2\sin x) = 0$

$2\cos x = 0 \qquad 2\sin x + 1 = 0$

$\cos x = 0 \qquad \sin x = -\frac{1}{2}$

$$\boxed{x = \frac{\pi}{3}, \frac{3\pi}{2} \qquad x = \frac{7\pi}{6}, \frac{11\pi}{6}}$$

$\theta' = \frac{\pi}{6}$



13. For θ in the interval $[0, 2\pi)$, what are the solutions of the equation below?

$$2\sin(3\theta) + 1 = 0$$

$0 \le \theta < 2\pi \qquad \text{Let } \alpha = 3\theta$

$0 \le 3\theta < 6\pi \qquad \frac{1}{3}\alpha = \theta$

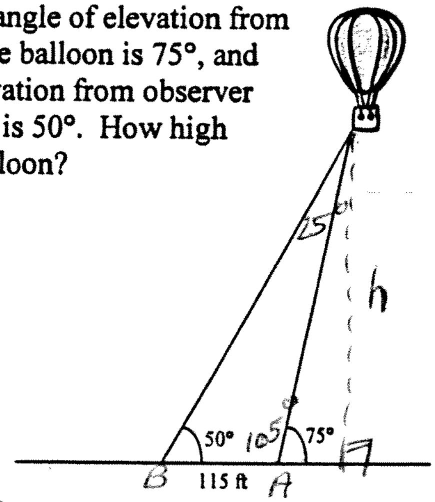
$2\sin\alpha + 1 = 0$

$\sin\alpha = -\frac{1}{2}$

$\left(\alpha = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}, \frac{31\pi}{6}, \frac{35\pi}{6}\right)$

$$\boxed{\theta = \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{31\pi}{18}, \frac{35\pi}{18}}$$

14. Observers at points A and B are directly in line with a hot-air balloon and are themselves 115 feet apart. The angle of elevation from observer A to the balloon is 75°, and the angle of elevation from observer B to the balloon is 50°. How high is the hot-air balloon?



$\frac{\sin 25^\circ}{115} = \frac{\sin 50^\circ}{b}$

$b = \frac{115\sin 50}{\sin 25}$

$b = 208.45$

$\sin 75 = \frac{h}{208.45}$

$208.45\sin 75^\circ = h$

$\boxed{201.35 = h}$

The ballon is about 201 ft high

Polynomial & Rational Functions (Chapter 3)

15. Determine if each of the functions below is ***polynomial***, ***rational***, or ***neither***.

polynomial a) $f(x) = -4x^2 - 9x$

neither b) $g(x) = 4^x + 5$

rational c) $h(x) = x^{-3} - 8$

neither d) $j(x) = x^{1/3} + 2$

16. What is the multiplicity of $x = ¼$ in $f(x) = 16x^4 - 24x^3 - 87x^2 + 47x - 6$?

$\frac{1}{4}$	16	-24	-87	47	-6
		4	-5	-23	6
$\frac{1}{4}$	16	-20	-92	24	0
		4	-4	-24	
$\frac{1}{4}$	16	-16	-96	0	
		4	-3		
	16	-12	-99		

$x = ¼$ has a multiplicity of 2.

17. Find ***all*** of the asymptote in the graph of $f(x) = \dfrac{2x+7}{x^2 - 10x - 24}$. $= \dfrac{2x+7}{(x-12)(x+2)}$

Horizontal Asymptote: $y = 0$

Vertical Asymptote(s): $x = 12,\ x = -2$

Oblique/Slant Asymptote: none

18. Find the ordered pair that describes the location of the removable discontinuity in the graph of $g(x) = \dfrac{8x-24}{x^2-9}$. $= \dfrac{8(x-3)}{(x+3)(x-3)}$

x-coordinate of hole is 3

y-coordinate $\dfrac{8}{3+3} = \dfrac{8}{6} = \dfrac{4}{3}$

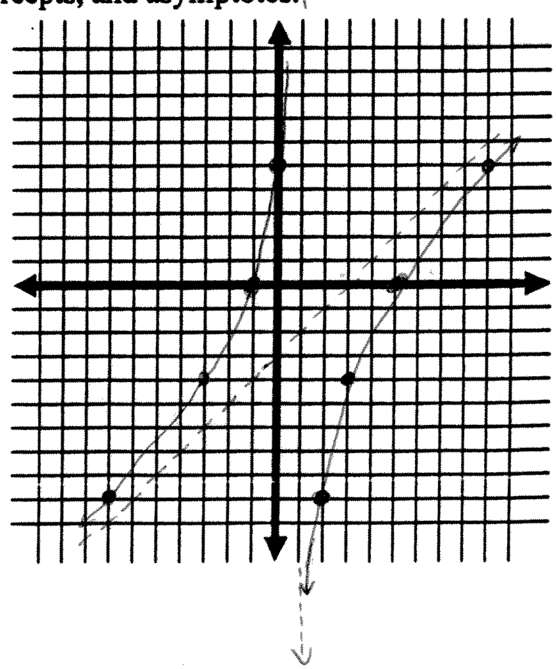
The hole in the graph is located at $\left(3, \frac{4}{3}\right)$.

19. Discuss the rational function by finding its domain, intercepts, and asymptotes.

$Q(x) = \dfrac{x^2 - 4x - 5}{x - 1}$

$$\begin{array}{r} x - 3 \\ x-1 \overline{) x^2 - 4x - 5} \\ -(x^2 - x) \\ \hline -3x - 5 \\ -(-3x + 3) \\ \hline -8 \end{array}$$

x	y
-7	-9
-3	-4
2	-9
3	-4
9	5



Domain: $\{x \mid x \neq 1\}$ or $(-\infty, 1) \cup (1, \infty)$

Range: $(-\infty, \infty)$ or $x \in \mathbb{R}$

Intercepts: (5, 0) (-1, 0) (0, 5)

Asymptote(s): VA: $x = 1$ OA: $y = x - 3$

Exponential & Logarithmic Functions (Chapter 4)

13. What is the solution of $4^{2x+7} = 64^{x-5}$? 2

$4^{2x+7} = 4^{3(x-5)}$

$2x+7 = 3x-15$

$22 = x$

14. What is the solution set of $\sqrt{x+5} = x - 7$? 2

$x+5 = x^2 - 14x + 49$

$0 = x^2 - 15x + 44$

$0 = (x-11)(x-4)$

$x = 11$ or $\cancel{x = 4}$

Check $x = 11$

$\sqrt{11+5} = 11-7$

$4 = 4$ ✓

Check $x = 4$

$\sqrt{9} \neq 4 - 7$

$3 \neq -3$

15. What is the solution of $e^{3x-2} = 12$? 2

$(3x-2)\ln e = \ln 12$

$3x - 2 = \ln 12$

$3x = 2 + \ln 12$

$x = \frac{1}{3}(2 + \ln 12)$

≈ 1.49

16. ***True*** or ***False***? (Please write a word for your answer.) 4

True a) $\log_8 24 - \log_8 3 = 1$

$\log_8 8 = 1$

True b) $\frac{1}{3}\log_2 4 = \log_2 \sqrt[3]{4}$

False c) $\log_6 7 = \dfrac{\log 7}{\log 7}$

False d) $\log_7 12 + \log_7 5 = \log_7 17$

17. What is the **domain** of $f(x) = \log_4 (x - 10)$? 2

$\{x \mid x > 10\}$

$D_f = (10, \infty)$

18. What is the ***sum*** of the solutions of $\log(15x - 36) = 2\log x$? 2

$\log(15x-36) = \log x^2$

$0 = x^2 - 15x + 36$

$0 = (x-12)(x-3)$

$x = 12$ or 3

The sum of the solutions is 15.

19. What is the solution set of $\log_6(x-9)+\log_6 x = 2$? 2

$\log_6(x^2-9x) = 2$

$x^2-9x = 6^2$

$x^2-9x-36=0$

$(x-12)(x+3)=0$

$x=12$ $\quad \cancel{x=-3}$

$\{12\}$

20. Solve. $\log_2\sqrt{x+13}=5$ 2

$2^5 = \sqrt{x+13}$

$32 = \sqrt{x+13}$

$1024 = x+13$

$1011 = x$

21. A cup of coffee, having a temperature of 180°F, is taken out of the microwave and placed in a room which is 70°F. After 4 minutes, the temperature of the coffee is 160°F. How long from the time the coffee is taken out of the microwave will it be until the temperature of the coffee is 130°F? 2

$160 = 70+(180-70)e^{k(4)}$

$90 = 110e^{4k}$

$\ln\frac{9}{11} = 4k$

$\frac{1}{4}\ln\frac{9}{11} = k$ $\quad k \approx -.0502$

$130 = 70+(110)e^{kt}$

$\frac{1}{k}\ln\frac{60}{110} = t$ $\quad t = 12.1$ min

22. The half-life of plutonium-241 is 13 years. If 1000 grams are present now, how much will be present in 30 years? 2

$500 = 1000e^{13k}$

$\ln\frac{1}{2} = 13k$

$\frac{1}{13}\ln\frac{1}{2} = k$ $\quad k \approx -.0533$

$A(30) = 1000e^{30(\frac{1}{13}\ln\frac{1}{2})}$

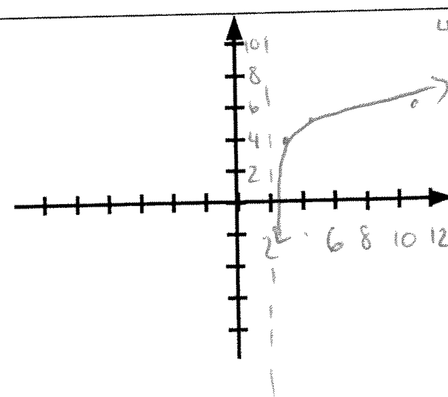
$A(30) = 201.983$ grams

23. Graph $f(x) = \log_3(x-2)+4$. (Show transformation tables.) 4

x	$y=3^x$
0	1
1	3
2	9

x	$\log_3 x$
1	0
3	1
9	2

x	$\log_3(x-2)+4$
3	4
5	5
11	6



Asymptote: $x=2$

Domain: $(2,\infty)$ Range: $(-\infty,\infty)$

12

Sequences, Series, & Binomial Theorem (Chapter 11)

24. What is the 5th term in the sequence defined by $a_1 = 12$ and $a_n = 2a_{n-1} + 3$? 2

$a_2 = 2(12) + 3 = 24 + 3 = 27$

$a_3 = 2(27) + 3 = 54 + 3 = 57$

$a_4 = 2(57) + 3 = 114 + 3 = 117$

$a_5 = 2(117) + 3 = 234 + 3 =$

237

25. Determine whether each of the following sequences is ***arithmetic***, ***geometric***, or ***neither***? 4

neither a) 3, 6, 11, 18, …

arithmetic b) 3, 7, 11, 15, …

neither c) 3, 11, 29, 66, …

geometric d) 3, 12, 48, 192, …

26. Find the following sums, if possible. 3

232 a) 57 + 49 + 41 + …+ 1 $d = -8$

$-56 = (n-1)(-8)$ | $1 = 57 + (n-1)(-8)$

$7 = n-1$ $\quad +56 = (n-1)8$ $\quad S_8 = \frac{8}{2}(57+1)$

$8 = n$ $\quad +7 = n-1$ $\quad = 4(58)$

$8 = n$ $\quad = 232$

$\frac{243}{2}$ b) 81 + 27 + 9 + 3 + … $r = \frac{1}{3}$

$S = \frac{81}{1-\frac{1}{3}} = \frac{81}{\frac{2}{3}}$

$S = 81(3/2)$

No Sum c) 4 + 12 + 36 + 108 + …

$r = 3$

27. What is the value of S_{14} for the series 17 + 8 + (-1) + (-10) + …? 2

$d = -9$

$S_{14} = \frac{14}{2}(17 + 17 + (14-1)(-9))$

$S_{14} = 7(34 + (13)(-9))$

$= 7(34 - 117)$

$= 7(-83)$

$= -581$

28. What is the value of $\sum_{k=1}^{7} 2(3)^k$? 2

$S_7 = 6\left(\frac{1-3^7}{1-3}\right)$

$S_7 = 6558$

29. Expresses the following series using summation notation. 2

1 + 4 + 9 + 16 + …

$\sum_{n=1}^{\infty} k^2$

30. What is the value of $\sum_{k=1}^{8}(k^2+5k+2)$? 2

$\frac{8(8+1)(2(8)+1)}{6} + 5\left(\frac{8(8+1)}{2}\right) + 2(8)$

$4(3)(17) + 5(4)(9) + 16$

$8+16+26+38+52+68+86+106$

400

31. What is the value of the 43rd term of the sequence -35, -29, -23, -17, …? 2

$a_1 = -35$

$d = 6$

$a_{43} = -35 + (43-1)(6)$

$a_{43} = 217$

32. What is the coefficient of x^6 in the expansion of $(x+2)^{11}$? 2

$\binom{11}{5} x^6 2^5$

$(462x^6)(32)$

$14,784 \cancel{x^6}$

33. You decide to deposit \$600 at the end of each month into an account that pays 7% interest compounded monthly to save for retirement. How much would be in the account after 20 years? 2

$A = 600\left(\frac{\left(1+\frac{.07}{12}\right)^{20\times12} - 1}{\left(\frac{.07}{12}\right)}\right)$

$A = 312,556$

34. Find a general formula for an arithmetic sequence whose 12th term is 35 and whose 34th term is 123. 2

$a_{12} = 35 \rightarrow 35 = a_1 + (12-1)d \rightarrow 35 = a_1 + 11d$

$a_{34} = 123 \rightarrow 123 = a_1 + (34-1)d \rightarrow 123 = a_1 + 33d$

$88 = 22d$

$4 = d$

$35 = a_1 + (11)(4)$

$35 = a_1 + 44$

$-9 = a_1$

$a_n = -9 + (n-1)(4)$ or $a_n = 4n - 13$

10

Limits & Derivatives (Chapter 13)

35. Use a table to evaluate $\lim_{x \to 0} \frac{2x^2 + x}{\tan x}$.   2

x	y
-.01	.97997
-.001	.998
-.0001	.9998
.0001	1.0002
.001	1.002
.01	1.02

$\lim_{x \to 0} \frac{2x^2+x}{\tan x} = 1$

36. ***True*** or ***False***? (Please write a word for your answer.)   4

True a) $\lim_{x \to 14} 42 = 42$

True b) $\lim_{x \to 28} x = 28$

False c) $\lim_{x \to 4} \frac{x^2 - 2x - 24}{x^2 - 7x + 12} = -2$ $\frac{(x-6)(x+4)}{(x-4)(x-3)}$

False d) $\lim_{x \to 5} \frac{2x^2 + 10x}{x^2 + 2x - 15}$ does not exist. $\frac{2x(x+5)}{(x+5)(x-3)}$

37. What is $\lim_{x \to 2} \frac{3x}{5x + 8}$?   2

$\frac{3(2)}{5(2)+8} = \frac{6}{18}$

$\frac{1}{3}$

38. What is $\lim_{x \to \infty} \frac{5x + 2}{4x + 3}$?   2

$\frac{5}{4}$

39. What is $\lim_{x \to 2} \frac{x^3 - 2x^2 + 9x - 18}{x^2 - 5x + 6}$?   2

$\lim_{x \to 2} \frac{(x^2+9)(x-2)}{(x-3)(x-2)} = \frac{(2)^2+9}{(2-3)} = \frac{13}{-1}$

$\lim_{x \to 2} \frac{x^3-2x^2+9x-18}{x^2-5x+6} = -13$

40. What is $\lim_{x \to 2} \frac{x^3 - 2x^2 + 9x - 18}{x^2 - x - 6}$?   2

$\lim_{x \to 2} \frac{(x^2+9)(x-2)}{(x-3)(x+2)} = 0$

41. What is the slope of the tangent line to the graph of $f(x) = x^2 - 7$ at (5, 18)?   2

$f'(x) = 2x$

$f'(5) = 10$

$m_{tan} = 10$

42. Find the equation of the line tangent to $f(x) = 4x^3 - x^2 - 3$ at (-1, -8).   2

$f'(x) = 12x^2 - 2x$

$f'(-1) = 12 + 2 = 14$

$y + 8 = 14(x+1)$

Find $\lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

43. Determine if $f(x)$ is continuous at $x = 2$. 3

$$f(x) = \begin{cases} x^2 + 7, & \text{if } x < 2 \\ 11, & \text{if } x = 2 \\ 5x + 1, & \text{if } x > 2 \end{cases}$$

① $f(2) = 11$ defined

② $\lim\limits_{x \to 2^-} = (2)^2 + 7 = 11$
$\lim\limits_{x \to 2^+} = 5(2) + 1 = 11$ } $\lim\limits_{x \to 2} f(x)$ exist

③ $\lim\limits_{x \to 2} f(x) = f(2)$

$f(x)$ is continuous at 2.

44. What is the derivative of $f(x) = \dfrac{4}{x-7}$? 2

$$f'(x) = \lim_{h \to 0} \frac{\frac{4}{x+h-7} - \frac{4}{x-7}}{h}$$

$$= \lim_{h \to 0} \frac{4(x-7)}{h(x+h-7)(x-7)} - \frac{4(x+h-7)}{h(x-7)(x+h-7)}$$

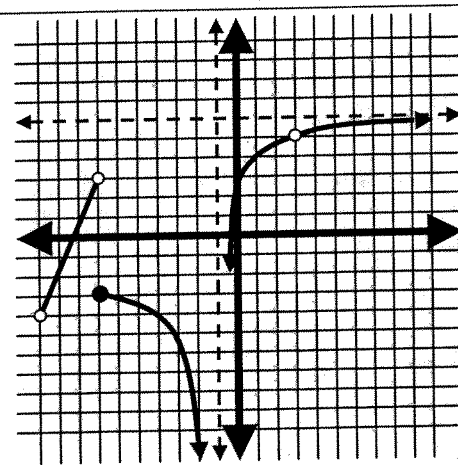
$$= \lim_{h \to 0} \frac{4x - 28 - 4x - 4h + 28}{h(x+h-7)(x-7)}$$

$$= \lim_{h \to 0} \frac{-4h}{h(x+h-7)(x-7)}$$

$$= \lim_{h \to 0} \frac{-4}{(x+h-7)(x-7)}$$

$$f'(x) = \frac{-4}{(x-7)^2}$$

45. Use the graph of $f(x)$ to find the following limits. 4



a) $\lim\limits_{x \to 3} f(x) = 5$

b) $\lim\limits_{x \to -7^+} f(x) = -3$

c) $\lim\limits_{x \to -1^-} f(x) = -\infty$

d) $\lim\limits_{x \to \infty} f(x) = 6$

Short Answer Use a phrase or a few sentences to answer the following questions.

46. How would you know if $x + 2$ is a factor of the polynomial function $P(x)$? 2

$P(-2) = 0$

or

The remainder in synthetic division is 0.

47. How would you determine if the graph of a rational function has a hole or a vertical asymptote for a specific value for which the rational function is undefined? 2

If the numerator & denominator factor, a hole will occur at value(s) for x for which the related factor cancels. The factors of vertical asymptotes do not cancel.

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Short Answer (continued) Use a phrase or a few sentences to answer the following questions.

48. What is the relationship between exponential functions and logarithmic functions? 2 Exponential and logarithmic functions are inverses of eachother	49. What conditions must be fulfilled in order for an infinite series to have a sum? 2 The series must be infinite geometric with a common ratio between -1 and 1, not inclusive.
50. What conditions must one show in order to prove a statement through the Principle of Mathematical Induction? 2 ① Show S_1 is true ② Show if S_k assumed to be true, S_{k+1} is true.	51. How would you know if a limit exists for a function $f(x)$ as x approaches c? 2 The limit as x approaches c from the left must equal the limit as x approaches c from the right.
52. What are the three conditions for a function to be continuous at $x = c$? 3 ① $f(c)$ is defined ② $\lim_{x \to c} f(x)$ exists ③ $\lim_{x \to c} f(x) = f(c)$	53. How are limits and end behavior related? 2 The end-behavior of a function may be described as a limit as x approaches $+\infty$ or $-\infty$

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Add **Bonus** You will see questions like this on the bonus section of your exam. For the purposes of the exam review, these questions are worth extra credit; therefore, they are optional.

B1. Solve on the interval [0°, 360°). Round your answer to the nearest degree. 2 4 tan x cos x + cos x = 0 $\cos x(4\tan x + 1) = 0$ $\cos x = 0$ or $\tan x = -\frac{1}{4}$ $x = 90°$ or $x = 270°$ $x = -14°$ $x = 180° - 14° = 166°$ $x = 360° - 14° = 346°$ 90°, 166°, 270°, 346°	B2. Solve. Round to the nearest hundredth. 2 $4^{3x} = 234$ $3x \ln 4 = \ln 234$ $x = \frac{\ln 234}{3 \ln 4}$ $x \approx 1.312$

B3. Express the series using summation notation. 2

$$\frac{6}{-8}+\frac{9}{1}+\frac{14}{10}+\ldots+\frac{54}{46}$$

$46=-8+(n-1)9 \quad a_n=-8+(n-1)9$

$54=(n-1)9 \quad =9n-17$

$6=n-1$

$7=n$

$$\sum_{k=1}^{7}\frac{k^2+5}{9k-17}$$

B4. Find the value of the limit, if it exists. 2

$$\lim_{x\to -4}\frac{\sqrt{x+29}-5}{x+4}\cdot\frac{\sqrt{x+29}+5}{\sqrt{x+29}+5}$$

$$=\lim_{x\to -4}\frac{x+29-25}{(x+4)(\sqrt{x+29}+5)}$$

$$=\lim_{x\to -4}\frac{1}{\sqrt{x+29}+5}$$

$$\frac{1}{10}$$

B5. Use the Principle of Mathematical Induction to prove S_n: $1+4+7+\ldots+(3n-2)=\frac{1}{2}n(3n-1)$. 2

$S_1: \frac{1}{2}(1)(3(1)-1) = \frac{1}{2}(2)=1 \quad S_1$ is True

Condition 2: S_k is true. Show S_{k+1} is true

$S_k: 1+4+7+\ldots+(3k-2)=\frac{1}{2}(k)(3k-1)$

$S_{k+1}: 1+4+\cancel{7}+\ldots+(3(k+1)-2) = \frac{1}{2}(k+1)(3(k+1)-1)$

$= \frac{1}{2}(k+1)(3k+2)$

$S_k + a_{k+1} \stackrel{?}{=} S_{k+1}$

$=\frac{1}{2}k(3k-1)+(3k-2+3)$

$=\frac{1}{2}k(3k-1)+(3k+1)$

$=\frac{3}{2}k^2-\frac{1}{2}k+3k+1$

$=\frac{3}{2}k^2+\frac{5}{2}k+1$

$=\frac{1}{2}(3k^2+5k+2) = \frac{1}{2}(3k+2)(k+1)$

$\therefore$ S_n is true through the Principle of Mathematical Induction for all positive numbers.

B6. Use the definition of the derivative to differentiate $f(x)=6x+\frac{5}{x}$. 2

$f(x)=6x+5x^{-1}$

$f'(x)=6-\quad x^{-2}$

$$f'(x)=\lim_{h\to 0}\frac{6(x+h)+\frac{5}{x+h}-(6x+\frac{5}{x})}{h}$$

$$=\lim_{h\to 0}\frac{6(x+h)(x)(x+h)}{x\,h(x+h)}+\frac{5x}{xh(x+h)}-\frac{5(x+h)}{(x)h(x+h)}$$

$$=\lim_{h\to 0}\frac{6x^2h+6xh^2+5x-5x-h}{x\,h(x+h)}$$

$$=\lim_{h\to 0}\frac{6x^2h+6xh^2-h}{x\,h(x+h)}$$

$$=\lim_{h\to 0}\frac{6x}{x+h}+\frac{6h}{x+h}-\frac{}{x(x+h)} = \frac{6x}{x}+\frac{6(0)}{x}-\frac{5}{x(x)} = 6-\frac{5}{x^2}$$