Objective: Students will be able to write evaluate piecewise defined functions, graph piecewise defined functions, evaluate the domain and range for piecewise defined functions, and solve application problems.

Notes: Piecewise Functions

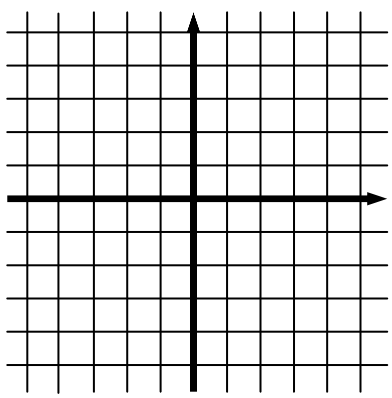
Piecewise-defined Function: a function that is defined differently for different parts of its domain. Pay attention to the domain description when evaluating and graphing.

Ex 1 Evaluate the following when $f(x)=\begin{cases}3x+4 & \text{if } -2\le x<2\\ 5 & \text{if } x=2\\ x^2-6 & \text{if } x>2\end{cases}$

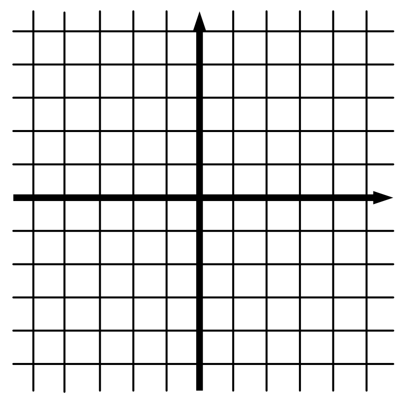
a) f(-1)	b) f(2)	c) f(4)	d) f(-4)

Ex 2 Graph the following piecewise-defined functions.

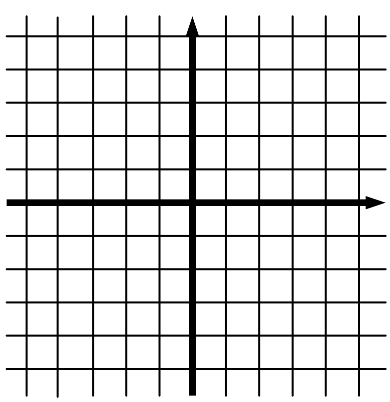
a) $f(x)=\begin{cases}3 & \text{if } x\le -2\\ 2x & \text{if } x>-2\end{cases}$



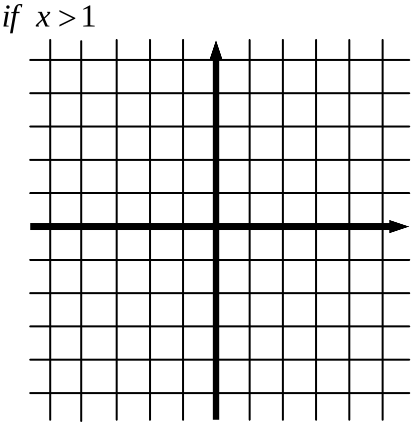
b) $g(x)=\begin{cases}x+2, & \text{if } x<3\\ x-1, & \text{if } x\ge 3\end{cases}$



c) $h(x)=\begin{cases}x^2-2, & \text{if } x<-1\\ -x, & \text{if } -1\le x\le 1\\ 3^x-4, & \text{if } x>1\end{cases}$



d) $j(x)=\begin{cases}-x-2 & \text{if } -4\le x<1\\ -4 & \text{if } x=1\\ x^2+1 & \text{if } x>1\end{cases}$



Ex 3 An economy car rented in Florida from National Car Rental® on a weekly basis costs \$95 per week. Extra days cost \$24 per day until the day rate exceeds the weekly rate, in which case the weekly rate applies. Find the cost C of renting an economy car as a piecewise function of the number x days used, where $7 \le x \le 14$. (Note: Any part of a day counts as a full day.)

You've Got Problems! Page 88 9-16, 29-38, 41, 43

WS - Piecewise Functions

Evaluate each of the following for the given function: $f(x)=\begin{cases} 70, & if\ -50<x\le -10 \\ x^2-9, & if\ -10<x\le 0 \\ 3x-8\ , & if\ 0<x\le 50 \end{cases}$

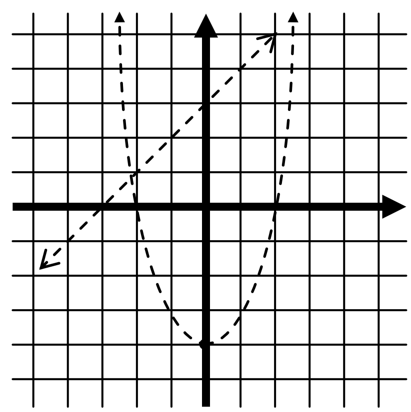
1. f(-20)
2. f(100)
3. f(30)
4. f(0)

5. What is the domain of f(x)?

6. What is the range of f(x)?

Each piece of the piecewise function is graphed with a dashed line without taking the domain description into account. Use the domain description to determine the location and type of endpoints and to make the final/complete graph of the piecewise function.

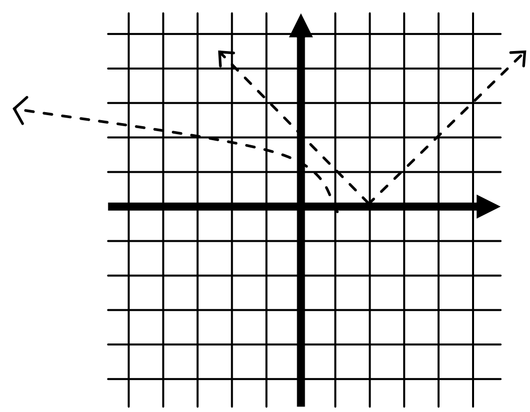
7. $g(x)=\begin{cases} x+3, & \text{if } x>\text{-}1 \\ x^2-4, & \text{if } x\le\text{-}1 \end{cases}$



D_g:

R_g:

8. $h(x)=\begin{cases} |x-2|, & \text{if } x\ge 0 \\ \sqrt{1-x}, & \text{if } x<0 \end{cases}$

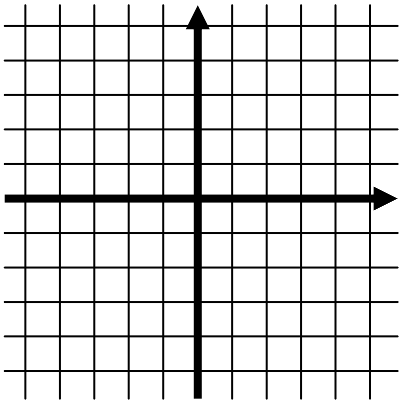


D_h:

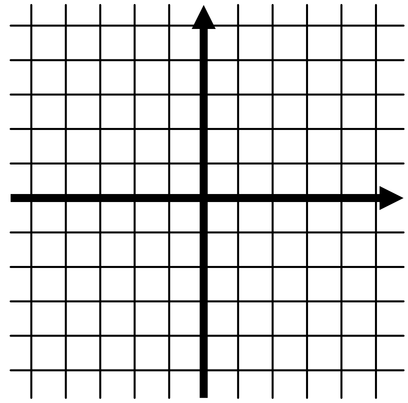
R_h:

Graph each piecewise function. Then, state each function's domain and range.

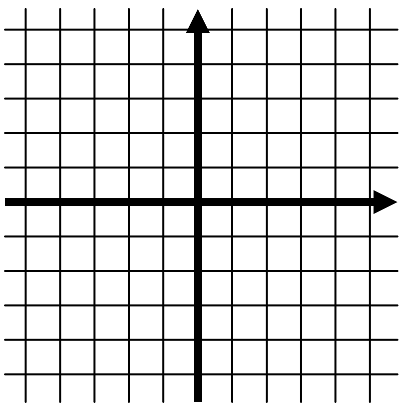
9. $f(x) = \begin{cases} -x+3, & \text{if } x > 0 \\ x-2, & \text{if } x \leq 0 \end{cases}$



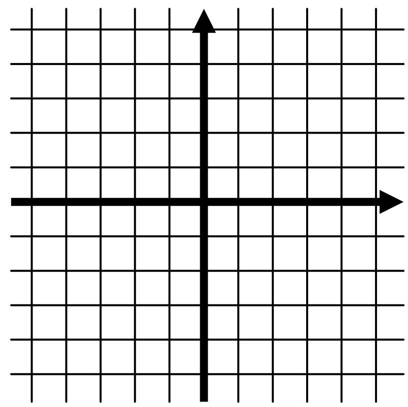
10. $g(x) = \begin{cases} \frac{1}{3}x+2, & \text{if } x \geq -3 \\ 2x+5, & \text{if } x < -3 \end{cases}$



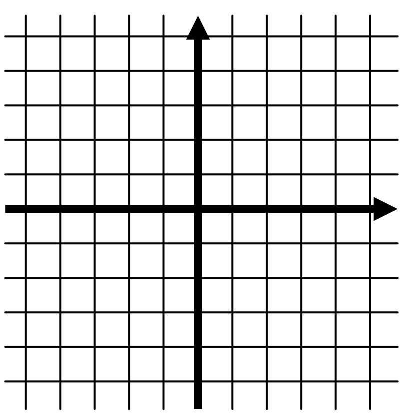
11. $h(x) = \begin{cases} -x, & \text{if } x < 1 \\ x^2, & \text{if } x \geq 1 \end{cases}$



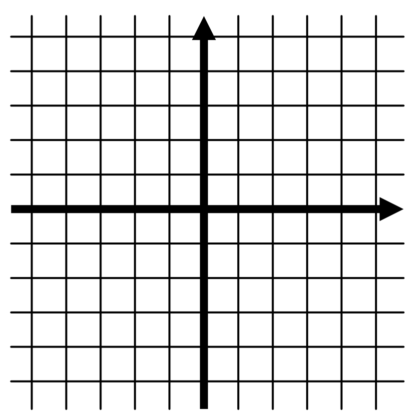
12. $j(x) = \begin{cases} -x+1, & x < 3 \\ -\sqrt{x-3}, & x \geq 3 \end{cases}$



13. $k(x) = \begin{cases} \sqrt{x+4}, & x < 2 \\ x+1, & x \geq 2 \end{cases}$



14. $p(x) = \begin{cases} -|x|-2, & \text{if } x < -1 \\ (x+3)^2, & \text{if } x \geq -1 \end{cases}$

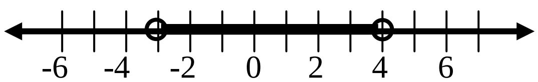
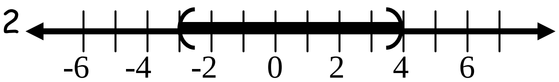


Objective: Students will be able to write interval notation, identify even and odd functions algebraically, and determine where a function is increasing, decreasing or constant

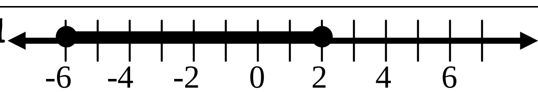
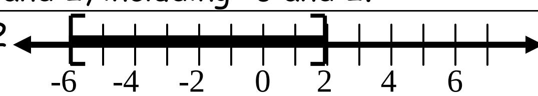
Notes: Interval Notation (domain and range)
And Properties of Functions

Interval Notation is a short way to describe all real numbers between two values.

Think about all of the real numbers between -3 and 4.

Graph 1 	Graph 2 
Set-builder Notation: $-3 < x < 4$	Interval Notation: $(-3, 4)$

Now, think about all of the real numbers between -6 and 2, including -6 and 2.

Graph 1 	Graph 2 
Set-builder Notation: $-6 \leq x \leq 2$	Interval Notation: $[-6, 2]$

Use interval notation to describe each statement.

______________________ 1. all of the real numbers between 5 and 12

______________________ 2. all of the real numbers between -3 and 11, including -3 and 11

______________________ 3. all of the real numbers between 50 and infinity

______________________ 4. all of the real numbers between 17 and infinity, including 17

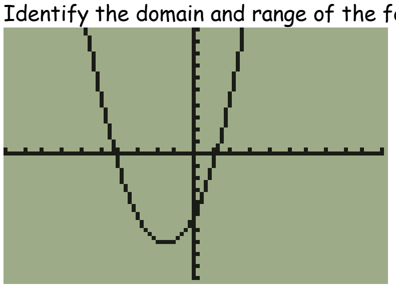
______________________ 5. all positive real numbers

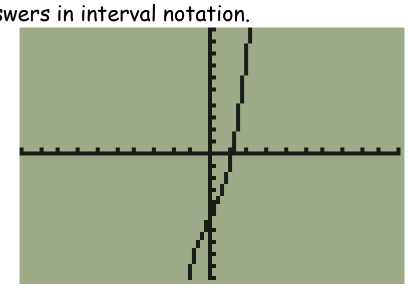
______________________ 6. all real numbers between negative infinity and 2

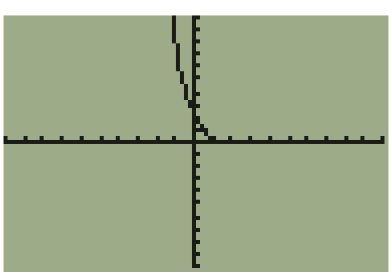
______________________ 7. all real numbers between negative infinity and 12 including 12

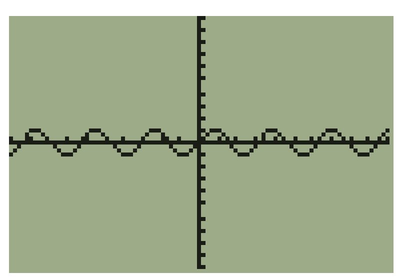
______________________ 8. all negative real numbers

Identify the domain and range of the following graphs. Write your answers in interval notation.









Even and Odd Functions: A function is…

- **even** if, for every x in the domain, –x is also in the domain and $f(-x) = f(x)$
- **odd** if, for every x in the domain, -x is also in the domain and $f(-x) = -f(x)$

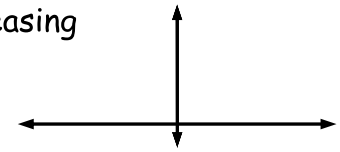
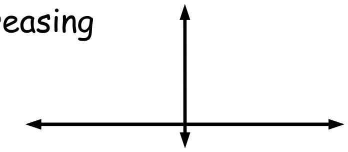
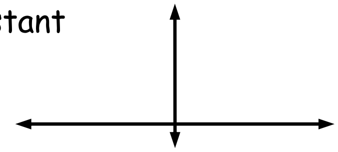
⋆**Even** functions have *y-axis* symmetry, and **odd** functions have *origin* symmetry.

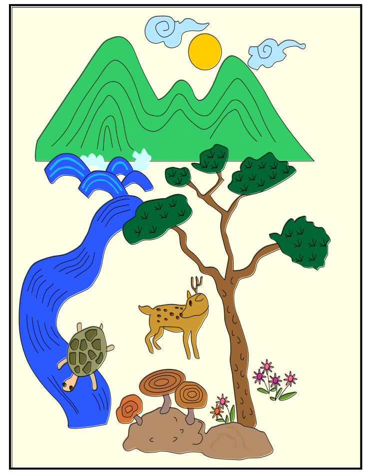
Ex 1 Determine if the following functions are *even*, *odd*, or *neither*.

a) $f(x) = x^3 - 2$	b) $g(x) = x^2 + 3$
c) $h(x) = \|x\|$	d) $F(x) = 4x^3 - x$

Increasing or Decreasing: Functions can increase, decrease or remain constant.

- A function is **increasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, $f(x_1) < f(x_2)$.
- A function is **decreasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, $f(x_1) > f(x_2)$.
- A function is **constant** on an open interval I if, for all choices of x in I, the values of f(x) are equal.

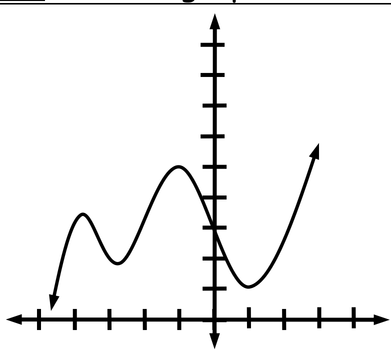
Increasing 	Decreasing 	Constant 



<u>Local Maximums and Local Minimums</u>: Bumps or dips in the graph of a function

- A function f has a **local maximum at c** if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(c) > f(x)$. We call $f(c)$ a *local maximum*.
- A function f has a **local minimum at c** if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(c) < f(x)$. We call $f(c)$ a *local minimum*.

<u>Ex 2</u> Use the graph of f to answer each question.

<table>
<tr><td rowspan="4"></td><td>a) When does f have a local maximum?</td></tr>
<tr><td>b) What are the local maxima?</td></tr>
<tr><td>c) When does f have a local minimum?</td></tr>
<tr><td>d) What are the local minima?</td></tr>
<tr><td colspan="2">e) When is f increasing?</td></tr>
<tr><td colspan="2">f) When is f decreasing?</td></tr>
</table>

<u>Average Rate of Change</u>:

If c is in the domain of f, the **average rate of change from c to x** is…

$\frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}, \quad x \neq c$	• It's essentially *slope*. • It's called the *difference quotient* in calculus.

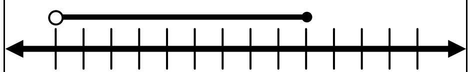
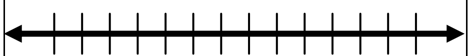
✯ The average rate of change of a function equals *the slope of the secant line* containing two points on its graph.

<u>Ex 3</u> Given f(x) = x^2 – 5…

a) find the average rate of change from 1 to 2	b) find the average rate of change from 1 to x	c) find the equation of the secant line containing (1, f(1)) and (3, f(3))

Worksheet 2.1
Functions

Complete the table below.

Graph	Interval Notation	Set Notation
1a) 	1b)	1c)
2a) 	2b) $(-\infty, 2) \cup [4, 7)$	2c)

Determine whether the equation is a function.

3. $y = \dfrac{1}{x}$	4. $y^2 = 4 - x^2$	5. $y = \|x\| + 3$

Given $f(x) = \dfrac{x^2 - 1}{x + 4}$**, find the following values or expressions.**

6. $f(0)$	7. $f(1)$	8. $f(-1)$

9. $f(-x)$	10. $-f(x)$
11. $f(2x)$	12. $f(x + 1)$

13. If $f(x) = 3x^2 + 2x - 4$, evaluate $\dfrac{f(x+h) - f(x)}{h}$.

Find the domain of each function.

14. $f(x) = \sqrt{3x-12}$	15. $g(x) = \dfrac{x}{x^2-16}$	16. $h(x) = \dfrac{x}{x^2+1}$

Given $f(x) = 3x + 4$ and $g(x) = 2x - 3$, find the following. Also, state the domain of the result.

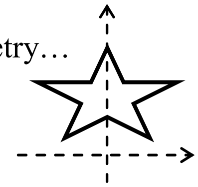
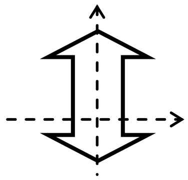
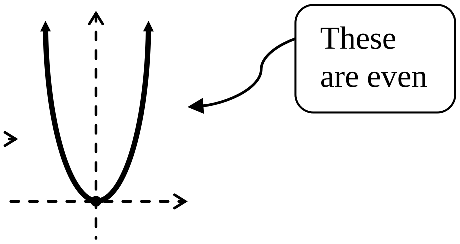
17. $f - g$ Domain:	18. $f \cdot g$ Domain:	19. $\dfrac{f}{g}$ Domain:

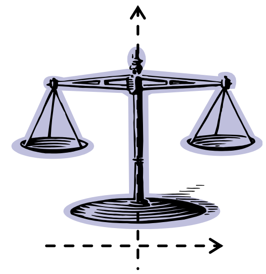
If a rock falls from a height of 20 meters on Earth, the height H (in meters) after x seconds is approximately $H(x) = 20 - 4.9x^2$.

20. What is the height of the rock when $x = 1.3$ seconds?	21. When is the height of the rock 10 meters?	22. When does the rock strike the ground?

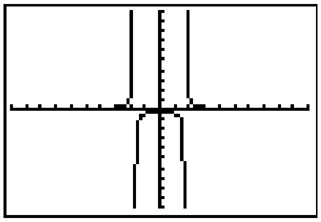
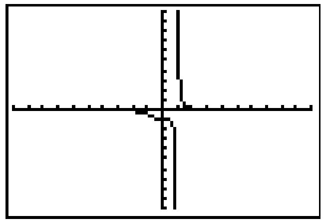
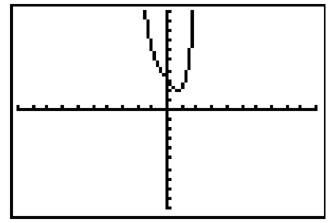
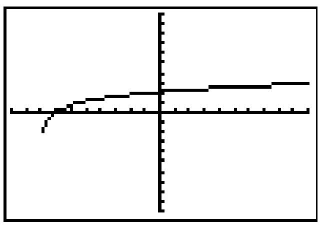
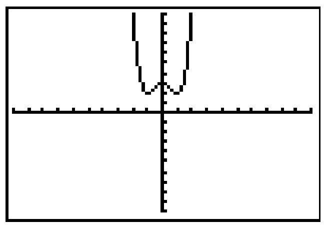
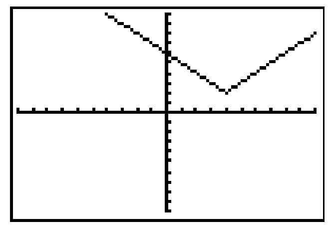
<u>Activity</u>: Even? Odd? Neither?

Remember y-axis symmetry…



Circle each even function.

Summary: All even functions …

Algebraic test for y-axis symmetry is…

1. substitute in –x
2. simplify
3. get the original function after simplifying

Example:

$f(x) = x^2 - 5$ is even because…

$f(-x) = (-x)^2 - 5$

$f(-x) = (-x)(-x) - 5$

$f(-x) = x^2 - 5$ This is the same as the original!

Box all of the even functions.

$f(x) = x^3 - x^2 + 2$

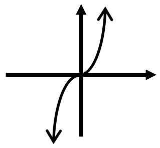
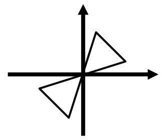
$h(x) = |x| - 8$

$g(x) = x^4 - 3x^2 + 5$

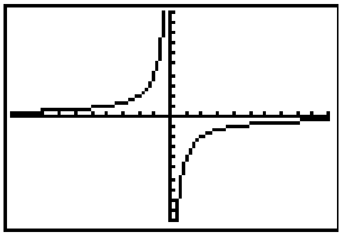
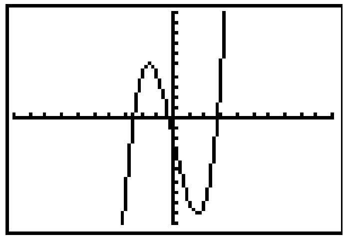
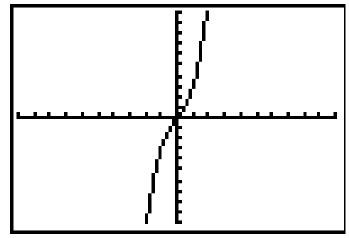
$j(x) = \sqrt{x} + 4$

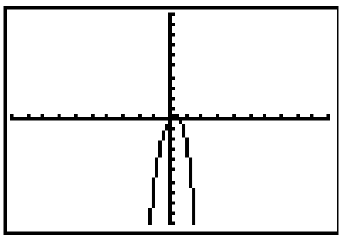
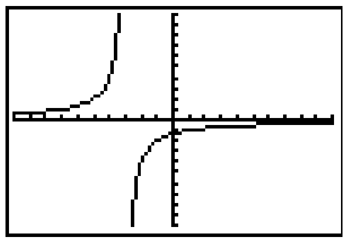
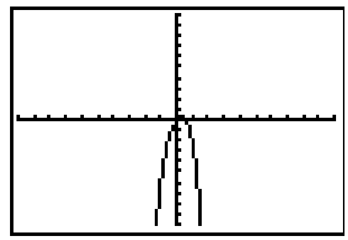
$m(x) = \dfrac{2}{x}$

$n(x) = \dfrac{1}{x^2 - 7}$

Remember origin symmetry…  

Circle each odd function.

Summary: All odd functions …

Example:

$f(x) = x^3 - 4x$ is odd because…

$(-y) = (-x)^3 - 4(-x)$

$-y = (-x)(-x)(-x) - 4(-x)$

$-y = -x^3 + 4$

$y = x^3 - 4$ ⇐ This is the same as the original!

Algebraic test for origin symmetry is…

1. substitute in –x and –y
2. simplify
3. get the original function after

Note: The algebraic test for odd functions doesn't plug in "–x" and "–y"; it only plugs in "-x," and uses the "–y" at the end, during the interpretation of the test. The final result may look like **-1·(original notation).**

Box all of the odd functions.

$f(x) = x^5 + x$

$g(x) = x^3 - 7$

$h(x) = \dfrac{3}{x}$

$j(x) = \dfrac{4}{x-5}$

$m(x) = \sqrt{x}$

$n(x) = \sqrt[3]{x}$

Objective: Students will be able to find a composite function and give the domain and range

Notes: Composite Functions

Composite Function: **Substituting** one function into another

- Notation: $(f \circ g)(x) = f((g(x))$
- The domain of $f \circ g$ is the set of all numbers x in the domain of g such that g(x) is in the domain of f.
  1. g(x) must be defined so that any x not in the domain of g must be excluded.
  2. f(g(x)) must be defined so that any x for which g(x) is not in the domain of f is excluded.
- Work from the right to the left for composition notation or inside to the outside for function notation.

Ex 1 Evaluate each expression using the values given in the table.

x	-3	-2	-1	0	1	2	3
f(x)	6	3	0	-3	-6	-9	-12
g(x)	-6	-2	-1	2	-1	-2	-6

a) $(f \circ g)(0) =$
b) $(g \circ f)(-1) =$
c) $(f \circ f)(-2) =$

Ex 2 Evaluate if $f(x) = 5x^2 - 4$ and $g(x) = 3x$

a) $(f \circ g)(1)$	b) $(g \circ f)(2)$	c) $(f \circ f)(-1)$	d) $(g \circ g)(4)$

Ex 3 Suppose $f(x) = x^2 - 3x + 8$ and $g(x) = 2x + 1$. Find the following composite functions. State the domain of each composite function.

a) $(f \circ g)(x)$	b) $(g \circ f)(x)$

Ex 4 If $f(x)=\frac{1}{x+5}$ and $g(x)=\frac{6}{x-2}$, find the domain of (f $\circ$ g)(x).

Domain of (f $\circ$ g)(x) is ____.

Try: Find the domain of (f $\circ$ g)(x) for the functions below.

1. $f(x)=\frac{4}{x+7}$ and $g(x)=\frac{3}{x-8}$	2. $f(x)=\frac{1}{3x-4}$ and $g(x)=\frac{2}{x^2-9}$

Ex 5 If $f(x)=\frac{1}{x+5}$ and $g(x)=\frac{6}{x-2}$, find the following compositions and their domains.

a) (f $\circ$ g)(x)	b) (g $\circ$ g)(x)
(f $\circ$ g)(x) = __________Domain:_________	(g $\circ$ g)(x) = __________Domain:_________

Objective: Students will be able to find an inverse, and verify if a function is one to one, both graphically and algebraically

Notes: Inverse Functions

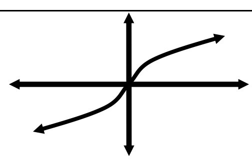
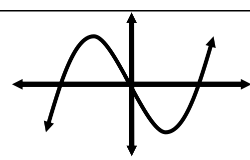
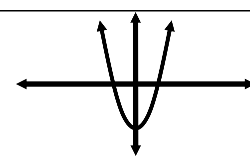
Inverse Functions: two functions that '**cancel**' each other out

- Notation: $\mathbf{f^{-1}}$ **or** $\mathbf{f^{-1}(x)}$
- **Switch** x's and y's
- **Domain of f(x) = Range of** $\mathbf{f^{-1}(x)}$ and **Domain** $\mathbf{f^{-1}(x)}$ **= Range of f(x)**
- The composition of f and its inverse is x. $f\left(f^{-1}(x)\right)=x$ and $f^{-1}\left(f(x)\right)=x$
- A function and its inverse are symmetric with respect to the line **y = x**
- A **one-to-one function** is a function in which different inputs never correspond to the same output. The inverse of a one-to-one function will be a function. *We must restrict some domains in order for some functions' inverses to be functions.*
- **Vertical-line Test** – A set of points in the x-plane is the graph of a function if and only if every vertical line intersects the graph in at most one point.
- The **horizontal line test** gives information about the graph of the inverse of a function. If a horizontal line passes through the graph of a function in at most **one** point, then the function is one-to-one. (Implication: The inverse of the function will be a function.)

Ex 1 Find the inverse of the functions below. Identify if the functions are one-to-one.

a) {(-2, 5), (-1, 2), (0, 1), (1, 2), (2, 5)} Inverse: One-to-one?	b) {(-2, -11), (-1, -4), (0, -3), (1, -2), (2, 5)} Inverse: One-to-one?

Ex 2 Analyze the following graphs to determine if the inverses will be functions.

a) 	b) 	c) 

Proving two functions are inverses of one another

1. Show that $f(g(x))=x$
2. Show that $g(f(x)) = x$
3. Write a sentence that justifies your conclusion

Ex 3 Prove that $f(x) = 2x - 5$ and $g(x) = \frac{1}{2}(x + 5)$ are inverses of each other by showing that $f(g(x)) = x$ and the $g(f(x)) = x$.

Finding the Inverse of a Function:

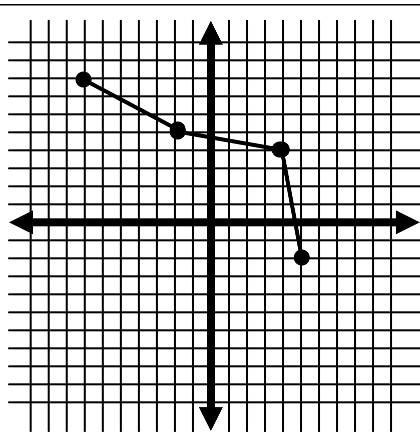
1. If f is not one-to one, define the domain of f so that f is one-to-one.
2. Switch the variables x and y to define f^{-1} implicitly.
3. Solve for y if possible to find the explicit form of f^{-1}.
4. Verify the result by showing that $f^{-1}(f(x)) = x$ and that $f(f^{-1}(x)) = x$.

Ex 4 Find the inverse of the following functions. State the domain and range of the function and its inverse.

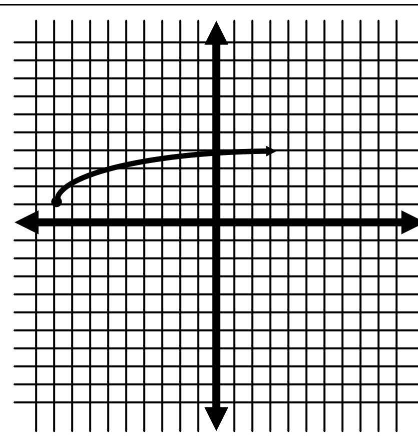
a) $f(x)=\frac{4x-1}{x+7}$		b) $g(x)=x^2+3$	
f(x) = Domain of f: Range of f:	$f^{-1}(x)$ = Domain of f^{-1}: Range of f^{-1}:	g(x) = Domain of g: Range of g:	$g^{-1}(x)$ = Domain of g^{-1}: Range of g^{-1}:

Ex 5 Graph the inverse of the functions in the graphs below.

a)



b)



Notice: The graphs are symmetric with respect to the line ________.