Notes: Piecewise Functions

Piecewise-defined Function: a function that is defined differently for different parts of its domain. Pay attention to the domain description when evaluating and graphing.

Ex 1 Evaluate the following when $f(x)=\begin{cases}3x+4 & \text{if } -2\le x<2 \\ 5 & \text{if } x=2 \\ x^2-6 & \text{if } x>2\end{cases}$

a) f(-1)	b) f(2)	c) f(4)	d) f(-4)
	$f(2)=5$		undefined

$f(-1)=3(-1)+4$
$f(-1)=-3+4$
$f(-1)=1$

$f(4)=(4)^2-6$
$f(4)=16-6$
$f(4)=10$

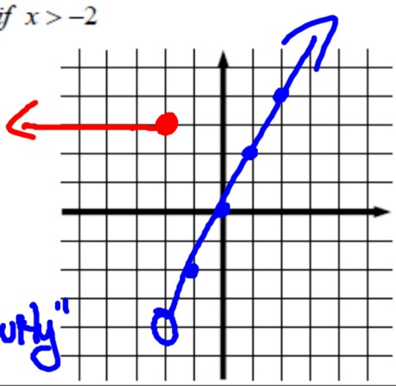
DNE
"Does Not Exist"

Ex 2 Graph the following piecewise-defined functions.

a) $f(x)=\begin{cases} 3 & \text{if } x\le -2 \\ 2x & \text{if } x>-2 \end{cases}$

$2(-2)=-4$

"Jump Discontinuity"



b) $g(x)=\begin{cases} x+2, & \text{if } x<3 \\ x-1, & \text{if } x\ge 3 \end{cases}$

$\lim\limits_{x\to 3^-} g(x)=5$

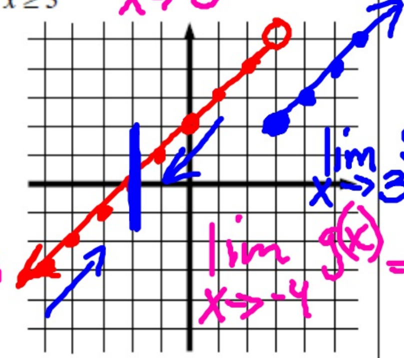
$3+2=5$

$(3)-1=2$

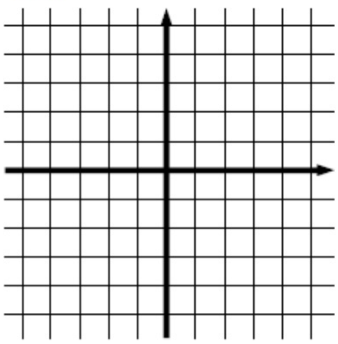
$\lim\limits_{x\to 3^+} g(x)$

$\lim\limits_{x\to -2} g(x)=0$

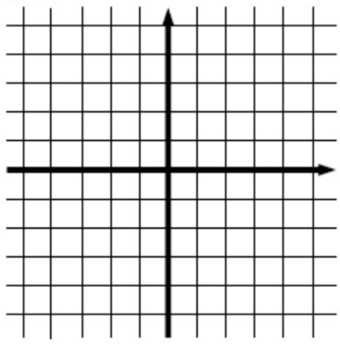
$\lim\limits_{x\to -4} g(x)=-2$



c) $h(x)=\begin{cases} x^2-2, & \text{if } x<-1 \\ -x, & \text{if } -1\le x\le 1 \\ 3^x-4, & \text{if } x>1 \end{cases}$



d) $j(x)=\begin{cases} -x-2 & \text{if } -4\le x<1 \\ -4 & \text{if } x=1 \\ x^2+1 & \text{if } x>1 \end{cases}$

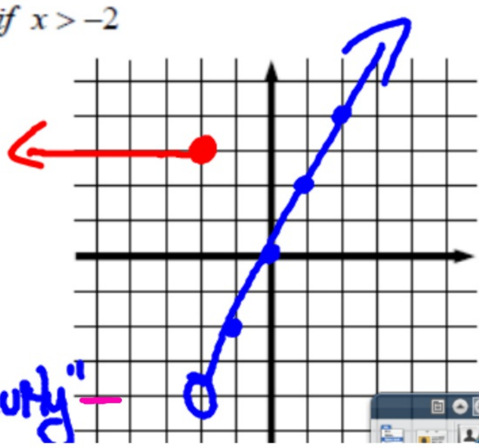


2 Graph the following piecewise-defined functions.

$$f(x)=\begin{cases}3 & \text{if } x\le -2\\ 2x & \text{if } x>-2\end{cases}$$

$(-2)=-4$

Jump Discontinuity"

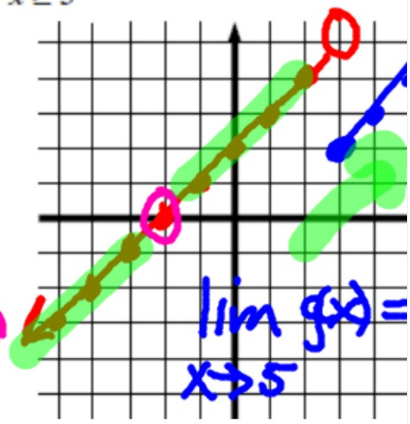


b) $$g(x)=\begin{cases}x+2, & \text{if } x<3\\ x-1, & \text{if } x\ge 3\end{cases}$$

$3+2=5$

$(3)-1=2$

$\lim_{x\to -2} g(x)=0$



$\lim_{x\to 5} g(x)=$

R: $(-4,\infty)$

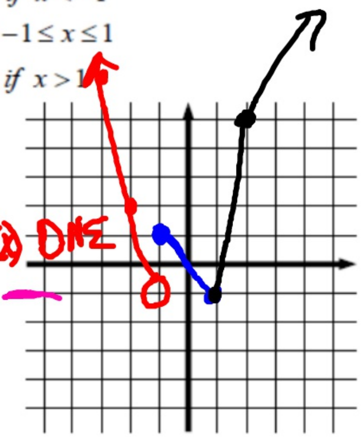
D: $(-\infty,\infty)$

$\lim_{x\to 3} g(x)$ DNE

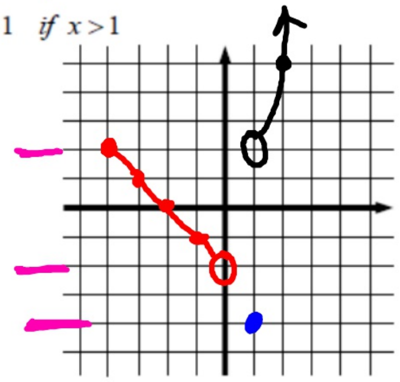
c) $h(x)=\begin{cases} x^2-2, & \text{if } x<-1 \\ -x, & \text{if } -1\le x\le 1 \\ 3^x-4, & \text{if } x>1 \end{cases}$

1	1
2	4
3	9

$\lim_{x\to -1} h(x)$ DNE



d) $j(x)=\begin{cases} -x-2 & \text{if } -4\le x< 0 \\ -4 & \text{if } x=1 \\ x^2+1 & \text{if } x>1 \end{cases}$



$h(-1)=(-1)^2-2$

$h(-1)=-1$

$h(-1)=-(-1)=1$

$h(1)=-(1)=-1$

$h(1)=3^1-4=-1$

$h(2)=3^2-4=5$

$\lim_{x\to 2^-} h(x)=5$

$j(-4)=-(-4)-2=2$

$j(0)=-(0)-2=-2$

$j(1)=(1)^2+1=2$

$j(2)=(2)^2+1=5$

Ex 3 An economy car rented in Florida from National Car Rental® on a weekly basis costs \$95 per week. Extra days cost \$24 per day until the day rate exceeds the weekly rate, in which case the weekly rate applies. Find the cost C of renting an economy car as a piecewise function of the number x days used, where $7 \le x \le 14$. (Note: Any part of a day counts as a full day.)

7 days → 0 → 7

Day 7 $95+24(0)$

Day 8 $95+24(1)$

Day 9 $95+24(2)$

Day 10 $95+24(3)$

Day 11 $95+\boxed{24(4)}$ $95=$ 190

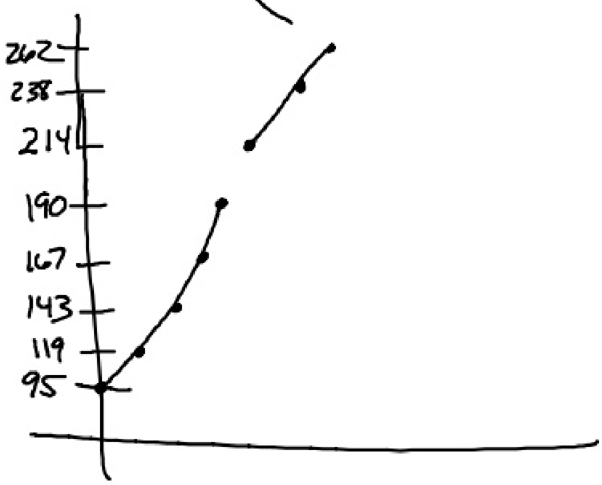
Day 12 ~~$95+24(5)$~~ $190+24(1)$

Day 13 ~~$95+24(6)$~~ $190+24(2)$

Day 14 ~~$95+24(7)$~~ $190+24(3)$

$$C(x)=\begin{cases} 95 \\ 95+24x \end{cases}$$

$$C(x)=\begin{cases} 95+24x & 0\le x\le 3 \\ 190+24x & 0\le x\le 3 \end{cases}$$



How do we write this as a piecewise function?

[redacted]

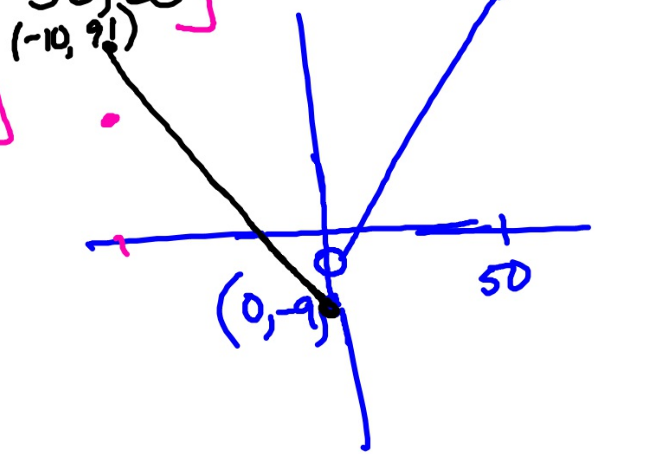
WS - Piecewise Functions

Evaluate each of the following for the given function: $f(x)=\begin{cases} 70, & \text{if } -50 < x \le -10 \\ x^2-9, & \text{if } -10 < x \le 0 \\ 3x-8, & \text{if } 0 < x \le 50 \end{cases}$

1. f(-20) 70
2. f(100) DNE
3. f(30) 82
4. f(0) = −9

5. What is the domain of f(x)? (−50, 50]

6. What is the range of f(x)? [−9, 142]



$f(-10)=70$

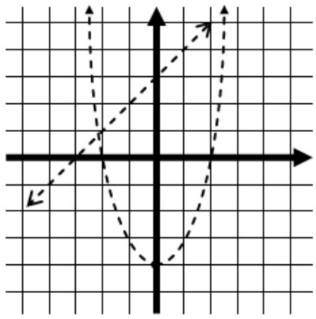
$f(-10)=(-10)^2-9=91$

$f(0)=-9$

$f(50)=142$

Each piece of the piecewise function is graphed with a dashed line without taking the domain description into account. Use the domain description to determine the location and type of endpoints and to make the final/complete graph of the piecewise function.

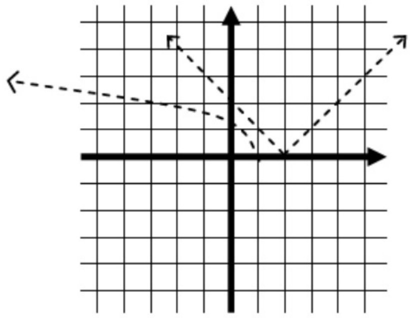
7. $g(x) = \begin{cases} x+3, & \text{if } x > -1 \\ x^2 - 4, & \text{if } x \leq -1 \end{cases}$



D_g:

R_g:

8. $h(x) = \begin{cases} |x-2|, & \text{if } x \geq 0 \\ \sqrt{1-x}, & \text{if } x < 0 \end{cases}$



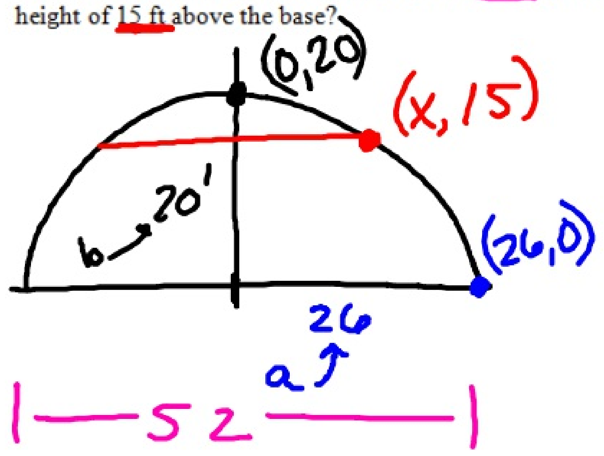
D_h:

R_h:

Application Problem

Include a drawing. Show all work that leads to your solution. Answer each question in a complete sentence. Be sure to include units in your final answers!

5. An arch is in the form of a semi-ellipse is 52 ft at the base and has a height of 20 ft. How wide is the arch at a height of 15 ft above the base?



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{26^2} + \frac{15^2}{20^2} = 1$$

$$\frac{x^2}{676} = 1 - \frac{225}{400}$$

$$\frac{x^2}{676} = \frac{400}{400} - \frac{225}{400}$$

$$\frac{x^2}{676} = \frac{175}{400}$$

$$x^2 = \left(\frac{175}{400}\right)676$$

$$x = \pm\sqrt{\left(\frac{175}{400}\right)(676)}$$

$$x \approx \pm 17.197$$

$\times 2$

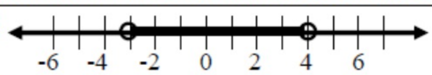
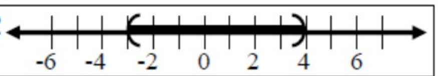
The arch is about 34 ft wide 15 ft above the base.

Objective: Students will be able to write interval notation, identify even and odd functions algebraically, and determine where a function is increasing, decreasing or constant

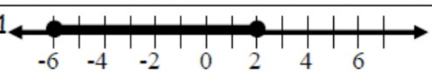
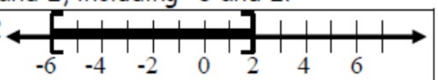
Notes: Interval Notation (domain and range)
And Properties of Functions

Interval Notation is a short way to describe all real numbers between two values.

Think about all of the real numbers between -3 and 4.

Graph 1 	Graph 2 
Set-builder Notation: $-3 < x < 4$	Interval Notation: $(-3, 4)$

Now, think about all of the real numbers between -6 and 2, including -6 and 2.

Graph 1 	Graph 2 
Set-builder Notation: $-6 \le x \le 2$	Interval Notation: $[-6, 2]$

Infinity and negative infinity are always soft brackets.

(   )

Use interval notation to describe each statement.

$(5, 12)$ 1. all of the real numbers between 5 and 12

$[-3, 11]$ 2. all of the real numbers between -3 and 11, including -3 and 11

$(50, \infty)$ 3. all of the real numbers between 50 and infinity

$[17, \infty)$ 4. all of the real numbers between 17 and infinity, including 17

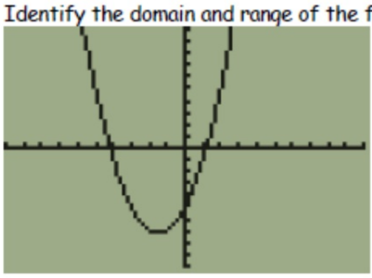
$(0, \infty)$ 5. all positive real numbers

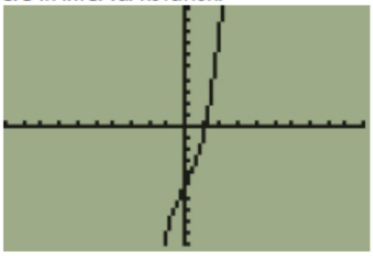
$(-\infty, 2)$ 6. all real numbers between negative infinity and 2

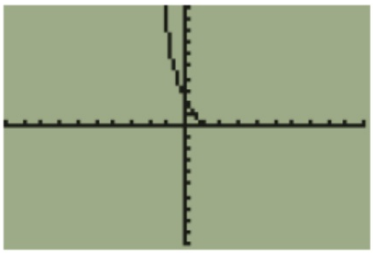
$(-\infty, 12]$ 7. all real numbers between negative infinity and 12 including 12

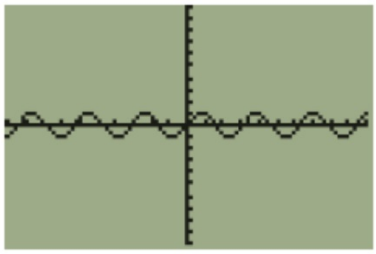
$(-\infty, 0)$ 8. all negative real numbers

Identify the domain and range of the following graphs. Write your answers in interval notation.

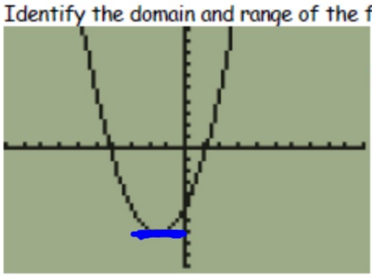






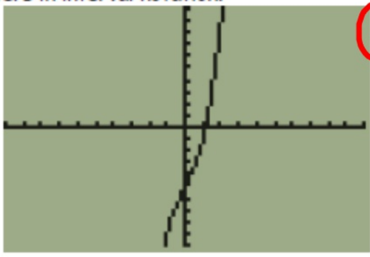


Identify the domain and range of the following graphs. Write your answers in interval notation.



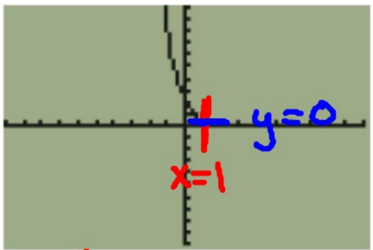
1 D: $(-\infty, \infty)$

R: $[-7, \infty)$



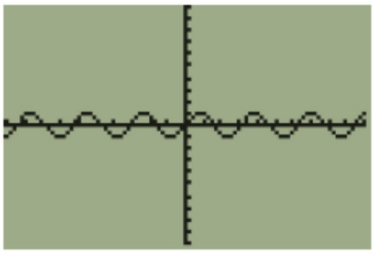
2 D: $(-\infty, \infty)$

R: $(-\infty, \infty)$



D: $(-\infty, 1]$

R: $[0, \infty)$



D: $(-\infty, \infty)$

R: $[-1, 1]$

1. Survey
2. WS – Piecewise Fcts, p.90
3. Watch vids on Even, odd, neither
4. PS due Fri

Even and Odd Functions: A function is...

- **even** if, for every x in the domain, -x is also in the domain and $f(-x) = f(x)$ EXACT
- **odd** if, for every x in the domain, -x is also in the domain and $f(-x) = -f(x)$

⋆**Even** functions have *y-axis* symmetry, and **odd** functions have *origin* symmetry.

Ex 1 Determine if the following functions are *even*, *odd*, or *neither*.

a) $f(x) = x^3 - 2$	b) $g(x) = x^2 + 3$
$-f(x) = -x^3+2$	Compare
$f(-x) = (-x)^3 - 2$	$g(-x) = (-x)^2 + 3$
$f(-x) = -x^3 - 2$	$g(-x) = x^2 + 3$
$-f(-x) = x^3 + 2$	Since $g(-x) = g(x)$, $g(x)$ is an even function.
Since $f(-x) \neq f(x)$ AND $f(-x) \neq -f(x)$, $f(x)$ is neither an even function nor an odd function.	

c) $h(x) = |x|$

$h(-x) = |-x| \Rightarrow |-1|\,|x|$

$h(-x) = |x|$

Since $h(-x) = h(x)$,

$h(x)$ is an even function.

d) $F(x) = 4x^3 - x$

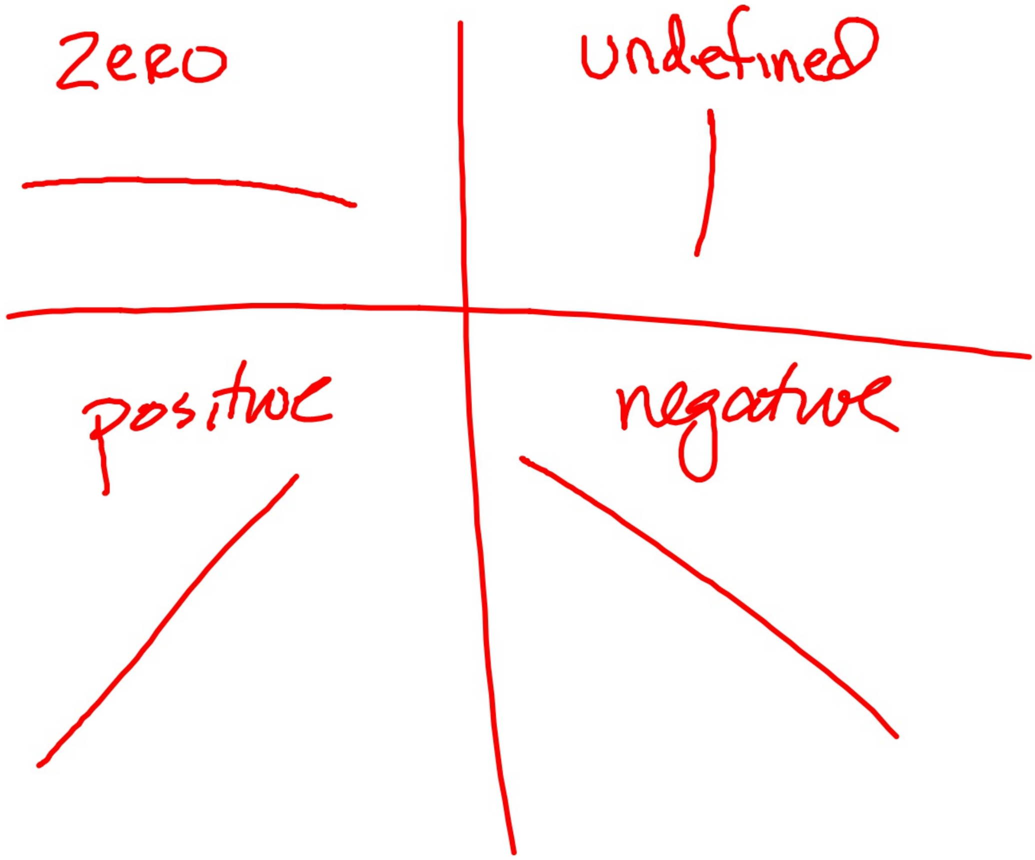
$F(-x) = 4(-x)^3 - (-x)$

$F(-x) = -4x^3 + x$

$-F(x) = -4x^3 + x$

Since $F(-x) = -F(x)$,

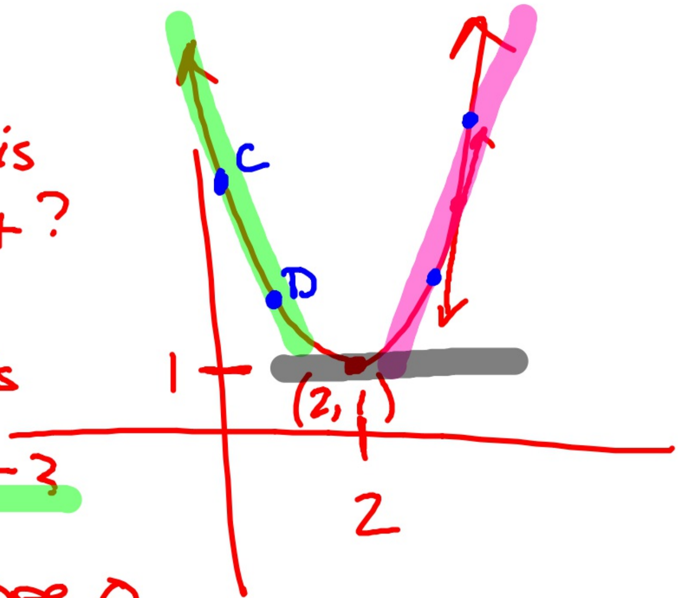
$F(x)$ is an odd function.

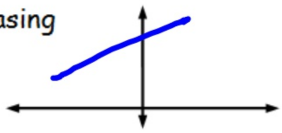
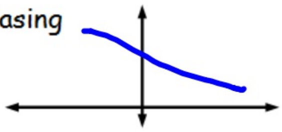
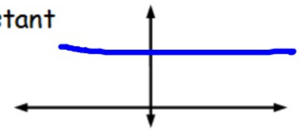

ZERO
undefined
positive
negative

① Where is slope + ?

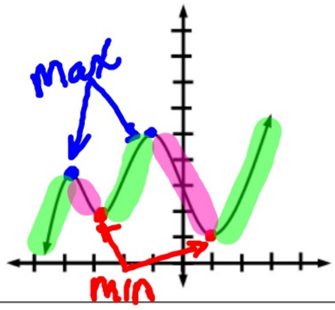
② Where is slope − ?

③ Is slope 0 anywhere?



Increasing	Decreasing	Constant
		

Ex 2 Use the graph of f to answer each question.



a) When does f have a local maximum?

$x = -1, \quad x = -4$

b) What are the local maxima?

$(-1, 5) \quad (-4, 3)$

c) When does f have a local minimum?

$x = -3 \quad x = 1$

d) What are the local minima?

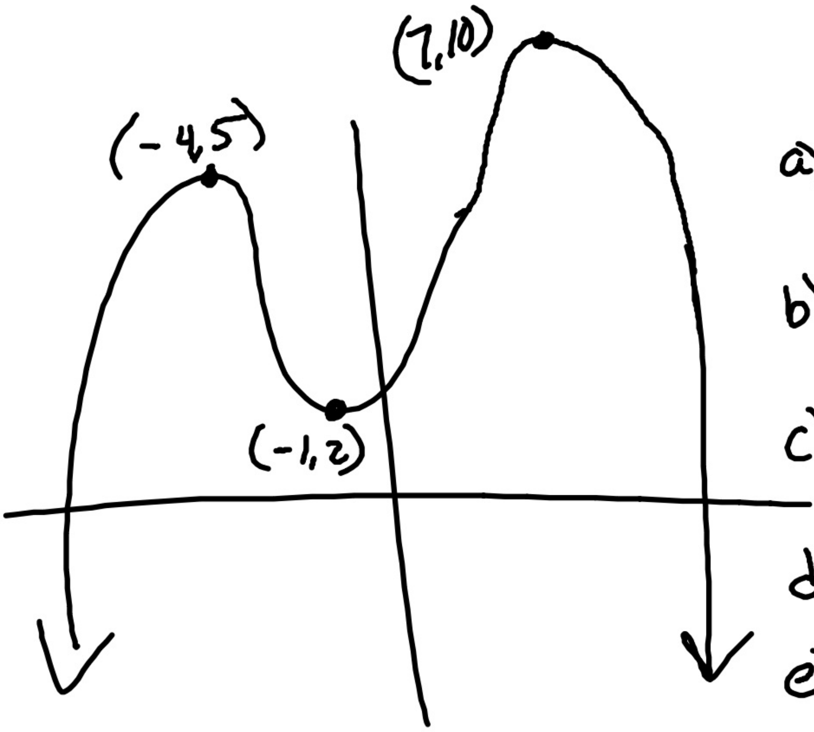
$(-3, 2) \quad (1, 1)$

e) When is f increasing?

$(-\infty, -4) \cup (-3, -1) \cup (1, \infty)$

f) When is f decreasing?

$(-4, -3) \cup (-1, 1)$



a) local max

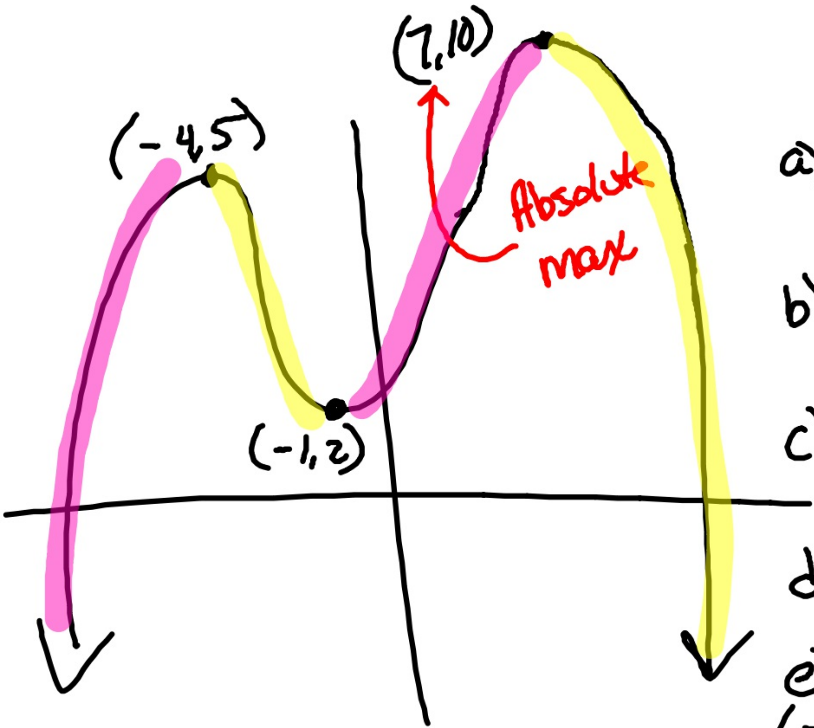
b)

c) local min

d)

e) ↑ intervals

f) ↓ intervals



a) local max $x = -4$

b) $(-4,5)$ absolute max $(7,10)$

c) local min $x = -1$

d) $(-1,2)$

e) ↑ intervals

$(-\infty,-4) \cup (-1,7)$

f) ↓ intervals

$(-4,-1) \cup (7,\infty)$

Average Rate of Change:

If c is in the domain of f, the **average rate of change from c to x** is...

$\dfrac{\Delta y}{\Delta x} = \dfrac{f(x) - f(c)}{x - c}, \quad x \neq c$	• It's essentially *slope.* • It's called the *difference quotient* in calculus.

★ The average rate of change of a function equals *the slope of the secant line* containing two points on its graph.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Ex 3 Given $f(x) = x^2 - 5$...

a) find the average rate of change from 1 to 2	b) find the average rate of change from 1 to x	c) find the equation of the secant line containing (1, f(1)) and (3, f(3))

$f(2) = (2)^2 - 5$

$f(2) = -1$

$f(1) = (1)^2 - 5$

$f(1) = -4$

$(2, -1)$

$(1, -4)$

$$\frac{\Delta y}{\Delta x} = \frac{f(2) - f(1)}{2 - 1} = \frac{(-1) - (-4)}{1} = 3$$

$(1, -4)$ $(3, 4)$

$f(3) = (3)^2 - 5 = 4$

$f(1) = (1)^2 - 5 = -4$

$$\frac{\Delta y}{\Delta x} = \frac{4 - (-4)}{3 - 1} = \frac{8}{2} = 4$$

c) find the equation of the secant line containing (1, f(1)) and (3, f(3))

$(1,-4)\quad(3,4)$

$f(3)=(3)^2-5=4$

$f(1)=(1)^2-5=-4$

$\frac{\Delta y}{\Delta x}=\frac{4-(-4)}{3-1}=\frac{8}{2}=4$

$y-y_1=m(x-x_1)$

$y-y_1=m(x-x_1)$

stop →

$\boxed{y-4=4(x-3)}$

$y-4=4x-12$

$y=4x-8$

$y-y_1=m(x-x_1)$

$y-(-4)=4(x-1)$

$\boxed{y+4=4(x-1)}$ stop

$y+4=4x-4$

$y=4x-8$

1 to x

$f(x) = (x)^2 - 5$

$f(x) = x^2 - 5$

$f(1) = (1)^2 - 5$

$f(1) = -4$

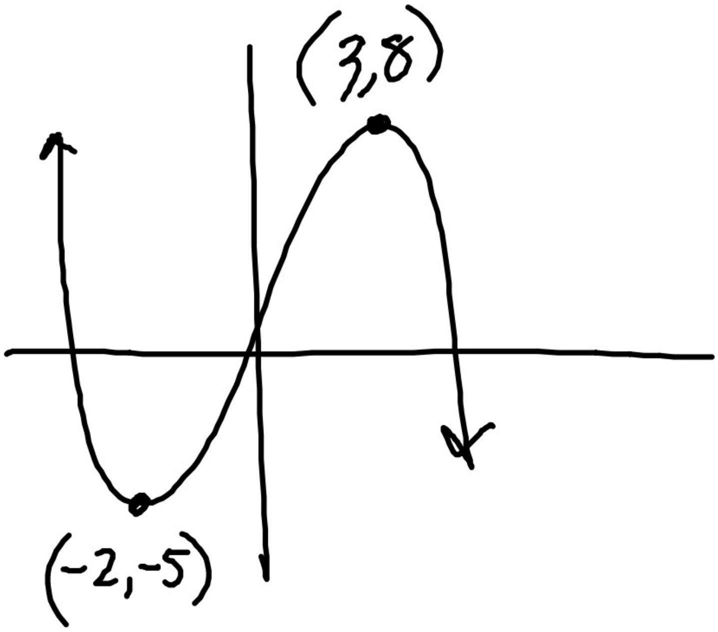
$$\frac{\Delta y}{\Delta x} = \frac{f(x) - f(1)}{x - 1}$$

$$\frac{\Delta y}{\Delta x} = \frac{(x^2 - 5) - (-4)}{x - 1}$$

$$\frac{\Delta y}{\Delta x} = \frac{x^2 - 1}{x - 1} = \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$\frac{\Delta y}{\Delta x} = x + 1$$

10/17-18



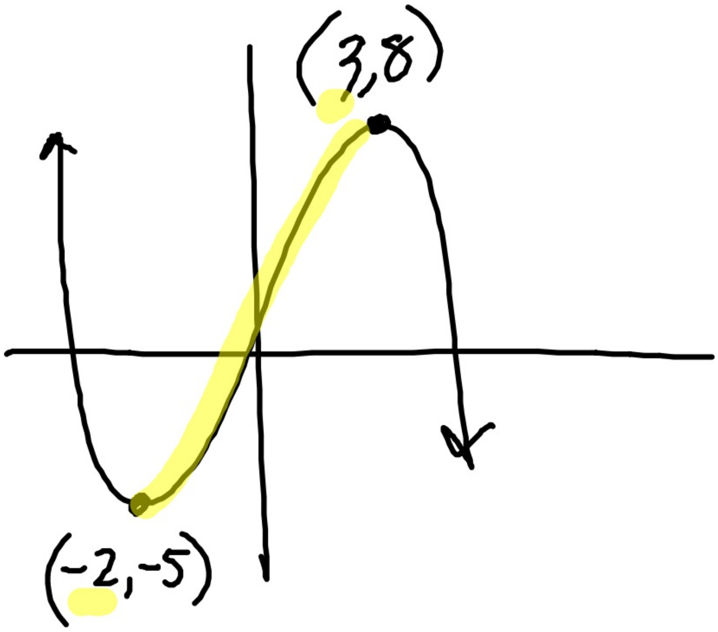
a) max

b) min

c) increasing intervals

d) decreasing intervals

10/17-18



a) max $x=3$

$(3,8)$

b) min $x=-2$

$(-2,-5)$

c) increasing intervals $(-2,3)$

d) decreasing intervals

$(-\infty,-2)\cup(3,\infty)$

Given: $f(x) = x^3 + 2x$, find the average rate of change from $x = -2$ to $x = 3$.

$$f(-2) = (-2)^3 + 2(-2)$$

$$f(-2) = -8 - 4$$

$$f(-2) = -12$$

$$f(3) = (3)^3 + 2(3)$$

$$f(3) = 33$$

$$\frac{f(3) - f(-2)}{3 - (-2)}$$

$$\frac{33 - (-12)}{5} = \frac{45}{5} = 9$$

Notes: Composite Functions

"fog" "gof"

Composite Function: Substituting one function into another

- Notation: $(f \circ g)(x) = f((g(x))$
- The domain of $f \circ g$ is the set of all numbers x in the domain of g such that g(x) is in the domain of f.
    1. g(x) must be defined so that any x not in the domain of g must be excluded.
    2. f(g(x)) must be defined so that any x for which g(x) is not in the domain of f is excluded.
- Work from the right to the left for composition notation or inside to the outside for function notation.

Ex 1 Evaluate each expression using the values given in the table.

x	-3	-2	-1	0	1	2	3
f(x)	6	3	0	-3	-6	-9	-12
g(x)	-6	-2	-1	2	-1	-2	-6

a) $(f \circ g)(0) =$ $f(g(0)) = f(2) = -9$

b) $(g \circ f)(-1) =$

c) $(f \circ f)(-2) =$

b) $g(f(-1)) = g(0) = 2$

c) $f(f(-2)) = f(3) = -12$

Ex 2 Evaluate if $f(x) = 5x^2 - 4$ and $g(x) = 3x$

a) $(f \circ g)(1) = 41$	b) $(g \circ f)(2)$	c) $(f \circ f)(-1)$	d) $(g \circ g)(4)$
$g(1) = 3(1) = 3$ $f(3) = 5(3)^2 - 4$		$f(-1) = 5(-1)^2 - 4$ $f(-1) = 1$ $f(f(-1)) = 5(1)^2 - 4$	

$\boxed{f(g(1)) = 41}$

$\boxed{f(f(-1)) = 1}$

b $f(2) = 5(2)^2 - 4$

$f(2) = 16$

$g(f(2)) = 3(16)$

$\boxed{g(f(2)) = 48}$

d $g(4) = 3(4) = 12$

$g(g(4)) = 3(12)$

$g(g(4)) = 36$

Ex 3 Suppose $f(x) = x^2 - 3x + 8$ and $g(x) = 2x + 1$. Find the following composite functions. State the domain of each composite function.

a) $(f \circ g)(x)$	b) $(g \circ f)(x)$

$f(2x+1) = (2x+1)^2 - 3(2x+1) + 8$

$f(2x+1) = 4x^2 + 4x + 1 - 6x - 3 + 8$

$f(2x+1) = 4x^2 - 2x + 6$

Domain: all real #'s

$(-\infty, \infty)$

$\mathbb{R}$

Ex 3 Suppose $f(x) = x^2 - 3x + 8$ and $g(x) = 2x + 1$. Find the following composite functions. State the domain of each composite function.

a) $(f \circ g)(x)$	b) $(g \circ f)(x)$

$g(x^2-3x+8) = 2(x^2-3x+8)+1$

$g(x^2-3x+8) = 2x^2-6x+16+1$

$g(x^2-3x+8) = 2x^2-6x+17$

Domain: $\mathbb{R}$ or $(-\infty,\infty)$

Ex 4 If $f(x)=\frac{1}{x+5}$ and $g(x)=\frac{6}{x-2}$, find the domain of $(f\circ g)(x)$.

$x\neq -5$ $\quad$ $x\neq 2$

$$f(g(x))=\frac{1}{\frac{6}{x-2}+5}=\frac{1}{\frac{6}{x-2}+\frac{5(x-2)}{1(x-2)}}$$

$$f(g(x))=\frac{1}{\frac{6}{x-2}+\frac{5x-10}{x-2}}=\frac{1}{\frac{5x-4}{x-2}}=\frac{x-2}{5x-4}$$

$$\boxed{f(g(x))=\frac{x-2}{5x-4}}$$

$5x-4\neq 0$

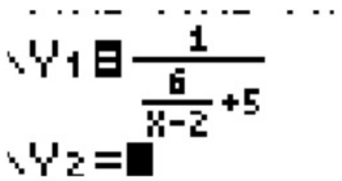
$5x\neq 4$

$x\neq 4/5$

Domain of $(f\circ g)(x)$ is $\{x \mid x\neq 4/5,\ x\neq 2\}$.

$(-\infty,4/5)\cup(\frac{4}{5},2)\cup(2,\infty)$

Y=

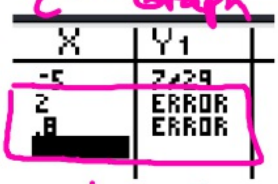


-on-

Y1(6/(x-2))+5)

2nd Window

2nd Graph



not in domain of f(g(x))

2. $f(x) = \frac{1}{3x-4}$ and $g(x) = \frac{2}{x^2-9}$

$$f(g(x)) = \frac{1}{3\left(\frac{2}{x^2-9}\right) - 4}$$

$$f(g(x)) = \frac{1}{\frac{6}{x^2-9} - \frac{4(x^2-9)}{x^2-9}}$$

$$f(g(x)) = \frac{1}{\frac{6-(4x^2-36)}{x^2-9}}$$

$$f(g(x)) = \frac{1}{\frac{-4x^2+42}{x^2-9}}$$

$$\boxed{f(g(x)) = \frac{1(x^2-9)}{-4x^2+42}} = \frac{(x+3)(x-3)}{-2(2x^2-21)}$$

Domain:

$x^2-9 \neq 0$

$x^2 \neq 9$

$x \neq \pm\sqrt{9}$

$x \neq \pm 3$

$x \neq -3,\ x \neq 3$

Domain $f(g(x))$

$-4x^2+42 \neq 0$

$-4x^2 \neq -42$

$x^2 \neq \frac{42}{4}$

$x \neq \pm\sqrt{\frac{42}{4}}$

$x \neq \pm\frac{\sqrt{42}}{2}$

$x \neq \pm 3.24$

$x \neq -3.24,$

$x \neq 3.24$

Domain:

$$\left(-\infty, -\frac{\sqrt{42}}{2}\right) \cup \left(-\frac{\sqrt{42}}{2}, -3\right) \cup (-3, 3) \cup \left(3, \frac{\sqrt{42}}{2}\right) \cup \left(\frac{\sqrt{42}}{2}, \infty\right)$$

$$\left\{x \mid x \neq \pm 3,\ x \neq \pm\frac{\sqrt{42}}{2}\right\}$$

$\boxed{\text{HW}}$ 1 PS

2 p. 229
#7, 11, 13, 25, 33, 67

3 Watch vids/take notes
on Inverse fcts
(p. 15-17 of notes)

Quiz 1

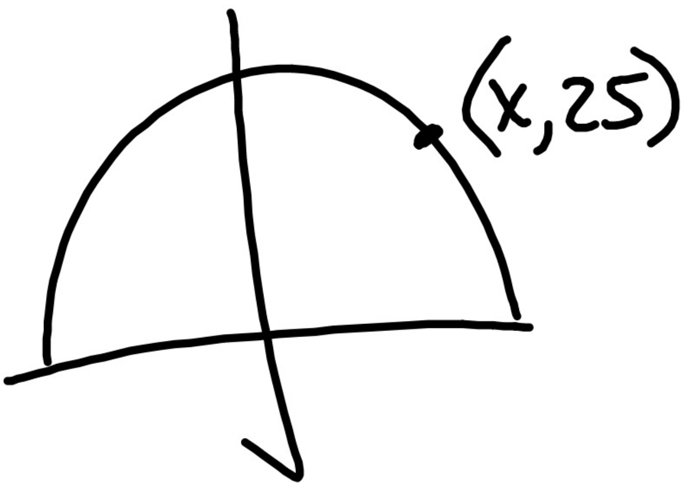
100 x .8

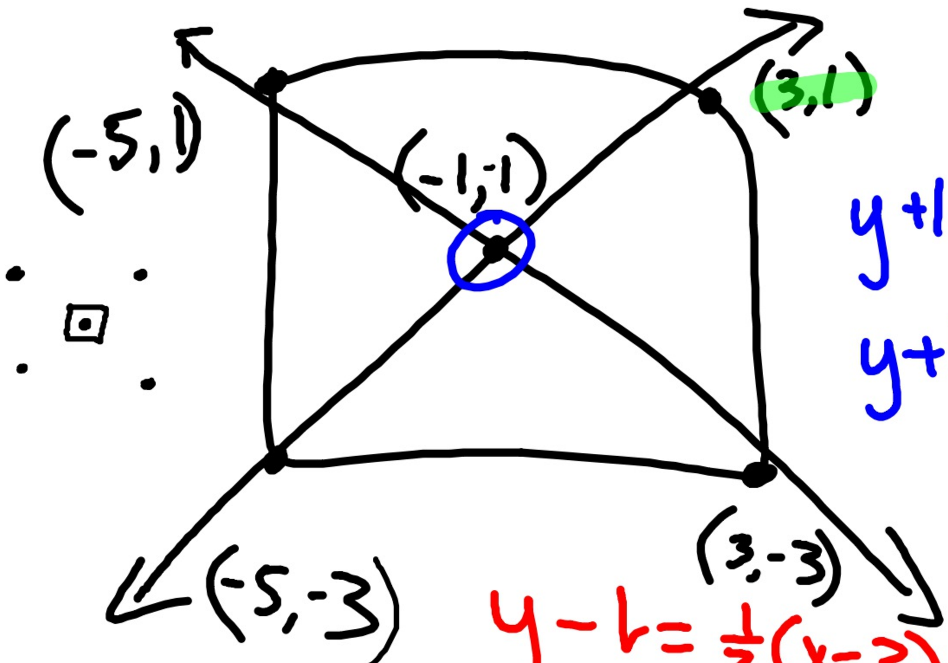
% · .80

90% · .80

$\boxed{72}$

Quiz 2





$$y+1=\frac{1}{2}(x+1)$$

$$y+1=-\frac{1}{2}(x+1)$$

$$y-1=\frac{1}{2}(x-3)$$